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AND LIMITS OF VISIBILITY FOR
DIFFERENTIAL CYLINDRICAL SURFACES
WITH PARALLEL GENERATING LINES

by Carol J. Sotos and Norbert O. Stockman

Lewis Research Center
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SUMMARY

The role of the view factor in radiation heat-transfer calculations is discussed. A general formula is derived for the view factor between differential strips of finitely or infinitely long general cylindrical surfaces having parallel generating lines. A comparison between the equation for finite length and that for infinite length is given. View factors using the general formula are presented for various pairs of differential strips of six different systems of circular cylinders and planes characteristic of space-radiator fin-tube geometries. The limits of visibility from each surface to all visible surfaces of each system are also presented.

Author

INTRODUCTION

Radiant heat transfer has received increased attention in recent years because of space applications and because of the use of higher operating temperatures in nuclear reactors, furnaces, turbines, and so forth. The only means of rejecting waste heat or controlling surface temperatures in space is by thermal radiation. In high-temperature systems, thermal radiation is involved as a major factor in the heat transfer within the system. There is a continual interchange of radiant energy between all surfaces that view each other. In many cases this interchange is a significant factor in the overall radiant heat transfer between the system and its environment. A discussion of the particular heat-transfer problems that arise in the study of space radiators can be found in reference 1.

In general, radiant interchange between two isothermal gray surfaces depends on the temperatures, the surface optical properties, and a geometrical relation between the surfaces. The geometrical relation is variously called view factor, configuration factor, angle factor, shape factor, and form factor. The term view factor will be used in this report.

The view factor from one surface (A_1) to another surface (A_2) is defined as that fraction of the total radiant energy leaving surface A_1 that strikes

surface A_2 . (Surfaces A_1 and A_2 are not necessarily separate surfaces; they may be parts of one surface that are visible to each other.) The general equation for the view factor from surface A_1 to surface A_2 involves a quadruple definite integral that is a function of the geometry of the surfaces and their relative orientation.

Most of the view-factor literature is concerned with evaluating the quadruple integral for various pairs of surfaces that are likely to occur in practice. This consists of two main steps: (1) formulating the integrand and limits of integration and (2) integrating the resulting expression. For some pairs of surfaces, both steps are readily accomplished and the result is a relatively simple formula. Several such formulas are given in reference 2. For other pairs of surfaces, either the formulation and the integration are extremely tedious or the resulting formulas are relatively complex so that approximate methods of evaluating the view factor are more practical.

Several approximate methods (numerical, mechanical, and optical) and extensive tabular and graphical results are presented and discussed in reference 2. Reference 3 presents formulations of the integrand and limits of integration for several pairs of surfaces likely to arise in heat-transfer analysis of spacecraft. These formulations are presented in such a way as to facilitate their incorporation into a computer program for numerical evaluation. View factors for circumferential elements of a pair of parallel tubes are given in reference 4. These view factors are useful for calculations of radiant interchange between longitudinally separated elements of parallel tubes that are circumferentially isothermal.

In reference 5, the specific formula is obtained for the view factor from an infinitely long differential strip on a tube to a similar strip on a fin parallel to the tube. This formula, which is applicable to the calculations for a particular fin-tube geometry with a centrally located fin, is only one of several (e.g., tube to tube) required to calculate the radiant interchange in a central-fin radiator.

A general formula is derived herein for the view factor between differential cylindrical strips. The general formula is used to obtain specific formulations of the differential view factors for all possible pairs of surfaces for a wide range of proposed space-radiator fin-tube configurations. The view factor formulas are given for strips of both finite and infinite length. A comparison between the formulas for the finitely and infinitely long strips is given as a function of a length parameter. This comparison can be used as an aid in determining a reasonable segment length when dividing a longitudinally nonisothermal radiator into longitudinal segments that are assumed to be isothermal. The limits of visibility for the various pairs of fin-tube surfaces are also formulated. These formulas for view factor and limits are also applicable to the heat-transfer calculations in other geometrically similar configurations, such as those that may occur in furnaces or reactors.

ANALYSIS

Formulation of Problem

A typical fin-tube radiator using a condensing vapor for space applications is shown in figure 1. The problem is to obtain an accurate prediction of the net heat rejected by such a radiator configuration so that an optimum, say minimum-weight, geometry can be found. The problem (fig. 2) includes the heat transferred from the fluid to the tube by condensation or convection, the conduction in the fin and tube,

and the radiation from the fin and tube surfaces. The radiation part of the problem includes the radiant interchange between fin and tube surfaces that are visible to each other and incident radiation, such as that which might come from the sun or planets.

The radiator of figure 1 is a direct-condensing radiator, which for design purposes, is essentially isothermal along the tube axis. The radiator geometry is a central fin-tube configuration. Other fin-tube geometries under consideration are shown in figure 3. For all geometries, the tubes are long compared to the distance between them; therefore, it is reasonable to assume that end effects can be neglected throughout most of the length. Since the tubes are long and isothermal, the radiator can be characterized by a two-dimensional cross section as shown in figure 3 and two-dimensional formulations of the heat-transfer equations can be used. Radiant interchange in the third dimension is taken care of by the view factor. When the radiator is not assumed to be infinite, the

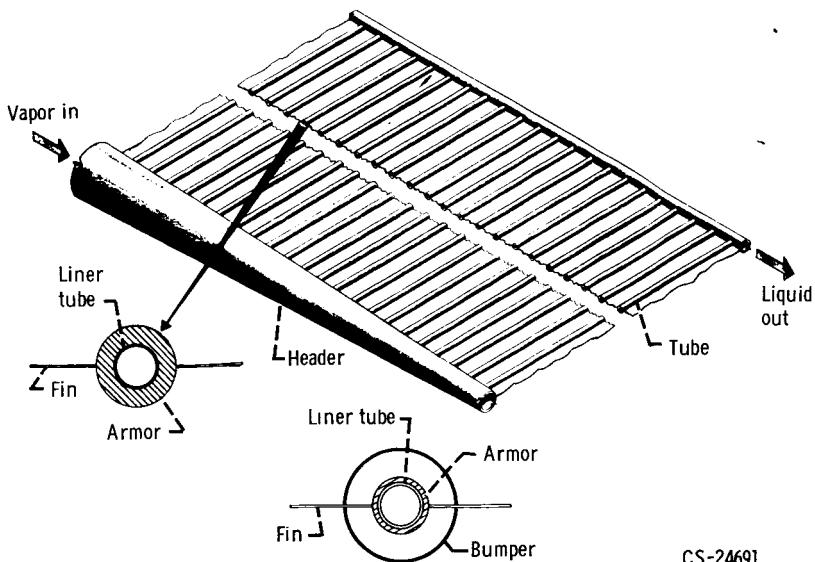


Figure 1. - Fin-tube radiator.

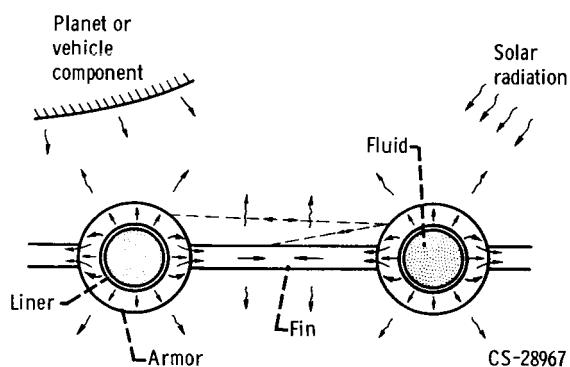


Figure 2. - Cross section of fin-tube radiator showing two-dimensional heat-transfer paths.

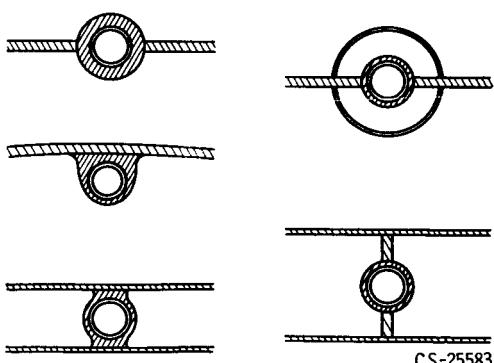


Figure 3. - Fin-tube geometries for space radiators.

actual length of the radiator (or a segment of the radiator as will be discussed in the section APPLICATIONS) appears in the view-factor formula.

This report is concerned only with the radiant interchange between the various fin and tube surfaces of the radiator and not with the conduction or convection that will also be part of the problem. In the following developments, the temperature at each point is assumed to be known. In actual radiator calculations, however, it will probably be obtained by solving the conduction and convection equations simultaneously with the radiation equations. For simplicity, the development is given in terms of gray surfaces.

The radiant interchange between elements of surfaces is formulated in terms of the radiosity, which is defined as the total radiant energy per unit time and per unit area, both emitted and reflected, leaving an element on a surface. (See ref. 6 for a discussion of radiosity.) The radiosity will be formulated for the three-dimensional case and will then be reduced to two dimensions by making appropriate assumptions. Consider first the general case of two cylindrical surfaces with parallel and equal-length generating lines as

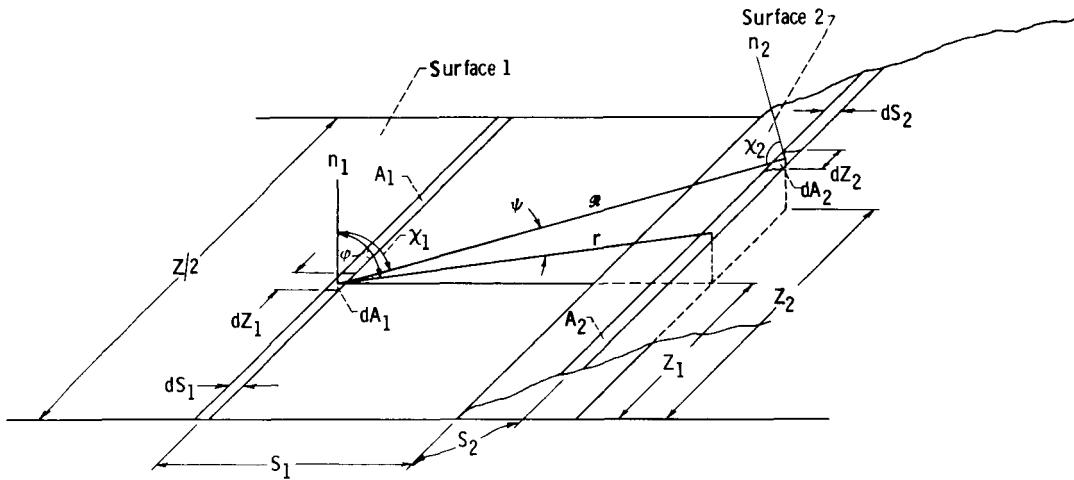


Figure 4. - Geometry for radiant interchange between two cylindrical surfaces with parallel and equal-length generating lines.

shown in figure 4. The radiosity $B(S_1, Z_1)$ leaving an element $dA_1 = dS_1 dz_1$ located at (S_1, Z_1) of surface 1 (fig. 4) is given by (All symbols are defined in the appendix.)

$$B(S_1, Z_1) = \epsilon \sigma T^4(S_1, Z_1) + \rho H(S_1, Z_1) \quad (1)$$

where ϵ is the emissivity of the surface, σ is the Stefan-Boltzmann constant, $T(S_1, Z_1)$ is the temperature of the surface at (S_1, Z_1) , ρ is the reflectivity of the surface, and $H(S_1, Z_1)$ is the total energy per unit time and per unit area incident upon dA_1 and is given by

$$H(S_1, Z_1) = \int_{S_{2l}(S_1)}^{S_{2u}(S_1)} \int_{-Z/2}^{Z/2} B(S_2, Z_2) dF_{dA_1-dA_2} + g(S_1, Z_1) \quad (2)$$

where $B(S_2, Z_2)$ is the radiosity from a differential element $dA_2 = dS_2 dZ_2$ of surface 2, $dF_{dA_1-dA_2}$ is the view factor from dA_1 to dA_2 which will be discussed in the section Derivation of General View Factors, $S_{2l}(S_1)$ is the lower limit of visibility of surface 2 as viewed from (S_1, Z_1) , $S_{2u}(S_1)$ is the upper limit of visibility of surface 2 as viewed from (S_1, Z_1) , and $g(S_1, Z_1)$ is the radiant energy per unit time and per unit area incident upon dA_1 from external sources such as the sun or planets. The limits $-Z/2$ and $Z/2$ are based on a total length Z of the surface and are constant. The limits of integration are functions only of S_1 and not of Z_1 .

Combining equations (1) and (2) yields

$$B(S_1, Z_1) = \epsilon\sigma T^4(S_1, Z_1) + \rho \int_{S_{2l}(S_1)}^{S_{2u}(S_1)} \int_{-Z/2}^{Z/2} B(S_2, Z_2) dF_{dA_1-dA_2} + \rho g(S_1, Z_1) \quad (3)$$

If surfaces 1 and 2 are assumed to be of constant temperature, constant radiosity, and constant incident radiation in the Z -direction, then equation (3) can be written

$$B(S_1) = \epsilon\sigma T^4(S_1) + \rho \int_{S_{2l}(S_1)}^{S_{2u}(S_1)} B(S_2) \int_{-Z/2}^{Z/2} dF_{dA_1-dA_2} + \rho g(S_1) \quad (4)$$

The integral $\int_{-Z/2}^{Z/2} dF_{dA_1-dA_2}$ is the view factor from element dA_1 to an elemental strip $Z dS_1$ and is written as $dF_{dA_1-dS_2}$. Putting $dF_{dA_1-dS_2}$ into equation (4) and integrating over Z_1 to get the radiosity per unit width per unit time $\int_{-Z/2}^{Z/2} B(S_1) dZ_1 = ZB(S_1)$ from strip $Z dS_1$ gives

$$ZB(S_1) = \epsilon\sigma ZT^4(S_1) + \rho \int_{S_{2l}(S_1)}^{S_{2u}(S_1)} B(S_2) \int_{-Z/2}^{Z/2} dF_{dA_1-dS_2} dZ_1 + \rho Zg(S_1)$$

Dividing by Z results in

$$B(S_1) = \epsilon\sigma T^4(S_1) + \frac{\rho}{Z} \int_{S_{2l}(S_1)}^{S_{2u}(S_1)} B(S_2) \int_{-Z/2}^{Z/2} dF_{dA_1-dS_2} dZ_1 + \rho g(S_1) \quad (5)$$

where the quantity $1/Z \int_{-Z/2}^{Z/2} dF_{dA_1-dS_2} dZ_1$ is usually written $dF_{dS_1-dS_2}$

and is the view factor from strip $Z dS_1$ to strip $Z dS_2$ and will be discussed in the section Derivation of General View Factors.

Putting $dF_{dS_1-dS_2}$ into equation (5) results in

$$B(S_1) = \epsilon\sigma T^4(S_1) + \rho \int_{S_{2l}(S_1)}^{S_{2u}(S_1)} B(S_2) dF_{dS_1-dS_2} + \rho g(S_1) \quad (6)$$

Equation (6) is two dimensional in that the quantities appearing in it are functions only of the two coordinates in the plane of the cross section and not of the longitudinal coordinates. However, the radiant interchange in the longitudinal direction is included in the view factor $dF_{dS_1-dS_2}$, and the total longitudinal length Z will appear in the various particular formulations of $dF_{dS_1-dS_2}$. Although the above development has been limited to two surfaces, any number of surfaces can be handled. For each additional surface another

integral, say $\rho \int_{S_{3l}(S_1)}^{S_{3u}(S_1)} B(S_3) dF_{dS_1-dS_3}$, would appear on the right side of

equation (6). The development has also been limited to gray surfaces. Nongray surfaces can be approximated by breaking up the energy spectrum into two or more regions with different gray surface properties for each region.

For a numerical solution of equation (6), the surfaces would be divided into small increments so that the condition of isothermal and constant-radiosity elements can be approximated. The differential view factors are assumed to apply to the small increments. As an illustration, consider the radiant interchange between one surface of the fin and the adjacent tubes of a central-fin configuration as shown in figure 5. Equation (6) written for increment $S_{i,j}^*$ of surface i of this figure would become

$$B(S_{i,j}^*) = \epsilon\sigma T^4(S_{i,j}^*) + \rho \sum_{k=1}^n \left[\sum_{j=j_l}^{j_u} B(S_{k,j}) dF_{S_{k,j}^*-S_{k,j}} \right] + \rho g(S_{i,j}^*) \quad (7)$$

where n is the number of surfaces visible to $S_{i,j}^*$, j_l is the number of the

increment of each surface corresponding to the lower limit of visibility from $S_{i,j}^*$, and j_u is the number corresponding to the upper limit. In particular, for increment $S_{1,4}^*$ on the left tube, equation (7) becomes

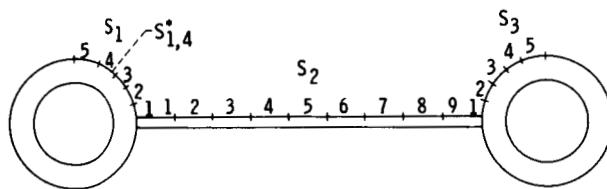


Figure 5. - Surface Increments for numerical solution.

$$B(S_{1,4}^*) = \epsilon \sigma T^4(S_{1,4}^*) + \rho \sum_{j=2}^9 B(S_{2,j}) dF_{S_{1,4}^* - S_{2,j}} + \rho \sum_{j=1}^5 B(S_{3,j}) dF_{S_{1,4}^* - S_{3,j}} + \rho g(S_{1,4}^*)$$

Equation (7) is written in a similar manner for each increment j of each surface i and this results in a number of equations in as many unknown B 's. In equation (7), however, the differential view factors and the limits of visibility for each pair of surfaces must first be formulated in terms of the geometry of each particular configuration.

Derivation of General View Factors

The view factor from one surface to another is a function only of the geometry of the two surfaces provided that the directional distribution (see ref. 2) of the radiation from the surface is diffuse. Diffuse radiation is distributed according to Lambert's cosine law, which states that the intensity of radiation in a given direction is proportional to the cosine of the angle between that direction and the normal to the surface. In this report, all surfaces are assumed to radiate diffusely.

The basic equation for the view factor from a point on a differential surface dA_1 to a differential surface dA_2 is (see, for example, ref. 7) the following:

$$dF_{dA_1 - dA_2} = \frac{1}{\pi} \cos \chi_1 d\omega \quad (8)$$

where χ_1 is the angle between the normal to dA_1 and the line joining dA_1 and dA_2 , $d\omega$ is the solid angle subtended at dA_1 by dA_2 , and, for the geometry of figure 6, is given by $d\omega = (\cos \chi_2) dA_2 / \pi R^2$. Substituting for $d\omega$ in equation (8) gives

$$dF_{dA_1 - dA_2} = \frac{\cos \chi_1 \cos \chi_2 dA_2}{\pi R^2} \quad (9)$$

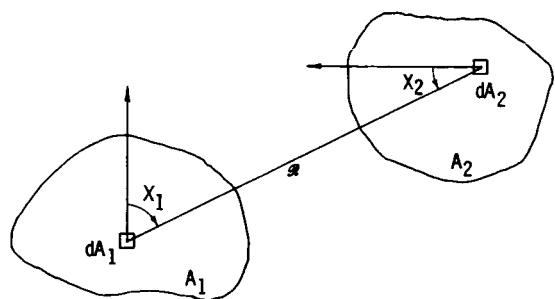


Figure 6. - Geometry for view factor between differential surfaces.

Equation (9) is the expression for the differential view factor most commonly found in the literature.

Consider now the case of a differential area dA_1 on a cylindrical surface and a differential area dA_2 that is located on a cylindrical surface whose

generating line is parallel to surface dA_1 (fig. 4, p. 4). Writing equation (8) in terms of the geometry of figure 4 gives

$$dF_{dA_1-dA_2} = \frac{1}{\pi} \cos \chi_1 d\omega_2 \quad (10)$$

where

$$\cos \chi_1 = \frac{r \cos \varphi}{\sqrt{r^2 + (z_2 - z_1)^2}}$$

and, as given in reference 8,

$$d\omega_2 = \frac{r d\varphi dz_2}{r^2 + (z_2 - z_1)^2} \cos \psi = \frac{r d\varphi dz_2}{r^2 + (z_2 - z_1)^2} \frac{r}{\sqrt{r^2 + (z_2 - z_1)^2}}$$

where r is the shortest distance between strips A_1 and A_2 and φ is the angle between r and the normal to A_1 . Substituting for $\cos \chi_1$ and $d\omega_2$, equation (10) becomes

$$dF_{dA_1-dA_2} = \frac{1}{\pi} \frac{r^3 \cos \varphi d\varphi dz_2}{[r^2 + (z_2 - z_1)^2]^2} \quad (11)$$

The view factor $dF_{dA_1-dS_2}$ from dA_1 to strip $A_2 = Z dS_2$ is obtained by integrating equation (11) from $z_2 = -Z/2$ to $z_2 = Z/2$

$$dF_{dA_1-dS_2} = \frac{r^3 \cos \varphi d\varphi}{\pi} \int_{-Z/2}^{Z/2} \frac{dz_2}{[r^2 + (z_2 - z_1)^2]^2} \quad (12)$$

Angle φ and distance r are measured in the cross section and are independent of Z_2 . Performing the integration in equation (12) and putting in the limits yield

$$dF_{dA_1-dS_2} = \frac{r^3 \cos \varphi d\varphi}{\pi} \frac{1}{2r^2} \left[\frac{\left(\frac{Z}{2} - z_1\right)}{r^2 + \left(\frac{Z}{2} - z_1\right)^2} + \frac{1}{r} \tan^{-1} \left(\frac{\frac{Z}{2} - z_1}{r} \right) \right. \\ \left. - \frac{\left(-\frac{Z}{2} - z_1\right)}{r^2 + \left(-\frac{Z}{2} - z_1\right)^2} - \frac{1}{r} \tan^{-1} \left(\frac{-\frac{Z}{2} - z_1}{r} \right) \right] \quad (13)$$

The view factor $dF_{dS_1-dS_2}$ from strip $A_1 = Z dS_1$ to strip $A_2 = Z dS_2$ is

obtained by taking the integrated mean of $dF_{dA_1-dS_2}$ over A_1

$$dF_{dS_1-dS_2} = \frac{1}{A_1} \int_{A_1} dF_{dA_1-dS_2} dA_1 \quad (14)$$

where

$$dA_1 = dZ_1 dS_1$$

Substituting for A_1 and dA_1 in equation (14) and canceling dS_1 , which is independent of Z_1 , result in

$$dF_{dS_1-dS_2} = \frac{1}{Z} \int_{-Z/2}^{Z/2} dF_{dA_1-dS_2} dZ_1 \quad (15)$$

Putting equation (13) into equation (15) and integrating yield

$$\begin{aligned} dF_{dS_1-dS_2} &= \frac{r \cos \varphi d\varphi}{2\pi Z} \left\{ -\frac{1}{2} \log \left[r^2 + \left(\frac{Z}{2} - Z_1 \right)^2 \right] + \frac{1}{2} \log \left[r^2 + \left(-\frac{Z}{2} - Z_1 \right)^2 \right] \right. \\ &\quad - \frac{\left(\frac{Z}{2} - Z_1 \right)}{r} \tan^{-1} \left(\frac{\frac{Z}{2} - Z_1}{r} \right) + \frac{1}{2} \log \left[r^2 + \left(\frac{Z}{2} - Z_1 \right)^2 \right] \\ &\quad \left. + \frac{\left(-\frac{Z}{2} - Z_1 \right)}{r} \tan^{-1} \left(\frac{-\frac{Z}{2} - Z_1}{r} \right) - \frac{1}{2} \log \left[r^2 + \left(-\frac{Z}{2} - Z_1 \right)^2 \right] \right\} \Big|_{-Z/2}^{Z/2} \end{aligned} \quad (16)$$

Equation (16) can be simplified to

$$\begin{aligned} dF_{dS_1-dS_2} &= \frac{\cos \varphi d\varphi}{2\pi Z} \left[-\left(\frac{Z}{2} - Z_1 \right) \tan^{-1} \left(\frac{\frac{Z}{2} - Z_1}{r} \right) \right. \\ &\quad \left. + \left(\frac{\frac{Z}{2} - Z_1}{r} \right) \tan^{-1} \left(\frac{\frac{Z}{2} + Z_1}{r} \right) \right] \Big|_{-Z/2}^{Z/2} \end{aligned} \quad (17)$$

Putting in the limits and simplifying result in

$$dF_{dS_1-dS_2} = \frac{\cos \varphi d\varphi}{\pi} \tan^{-1} \left(\frac{Z}{r} \right) \quad (18a)$$

or, equivalently,

$$dF_{dS_1-dS_2} = \frac{d(\sin \varphi)}{\pi} \tan^{-1}\left(\frac{Z}{r}\right) \quad (18b)$$

Equation (18b) is the formula for the view factor between differential strips of length Z on cylindrical surfaces whose generating lines are parallel to each other. Angle φ and distance r are functions of the geometry of the surfaces and the coordinates of the strips in a cross section perpendicular to the length Z . When Z is infinite, equation (18b) becomes

$$dF_{dS_1-dS_2}(\infty) = \frac{d(\sin \varphi)}{2} \quad (19)$$

This is the equation used in reference 5 to obtain the view factor from an infinite strip on a tube to an infinite strip on a fin. To facilitate comparison of equations (18b) and (19), the ratio

$$\frac{dF_{dS_1-dS_2}}{dF_{dS_1-dS_2}(\infty)} = \frac{\tan^{-1}\left(\frac{Z}{r}\right)}{\frac{\pi}{2}} \quad (20)$$

is plotted against Z/r in figure 7. It can be seen from figure 7 that, for a value of Z/r greater than about 10 the difference between the infinite and finite cases is less than five percent. On the other hand, if Z/r is less than 1, the difference between the infinite and finite is greater than 50 percent.

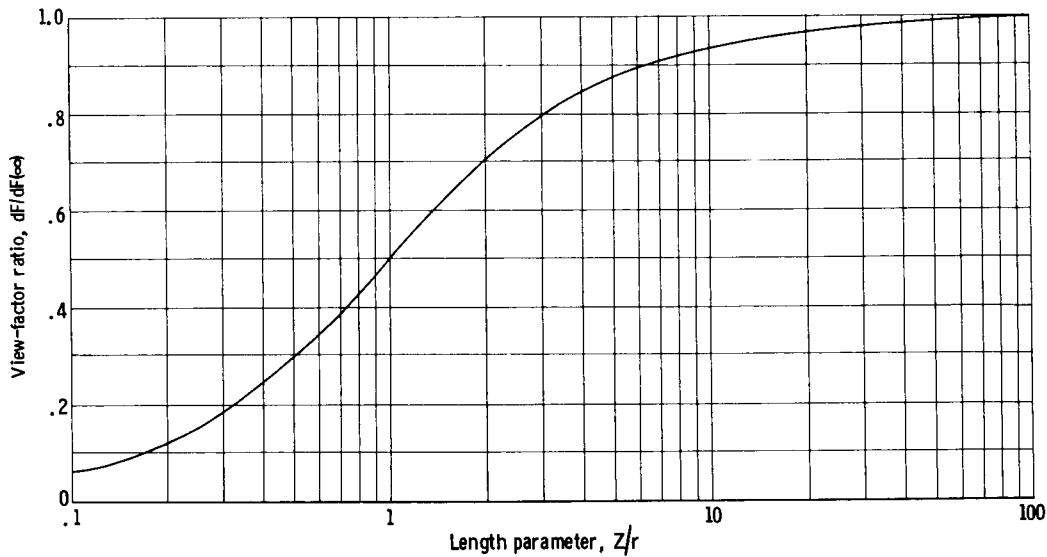


Figure 7. - View-factor ratio as a function of length parameter.

APPLICATIONS

The view factors presented herein are derived for several systems of parallel fins and tubes shown in figure 8. These are applicable primarily for

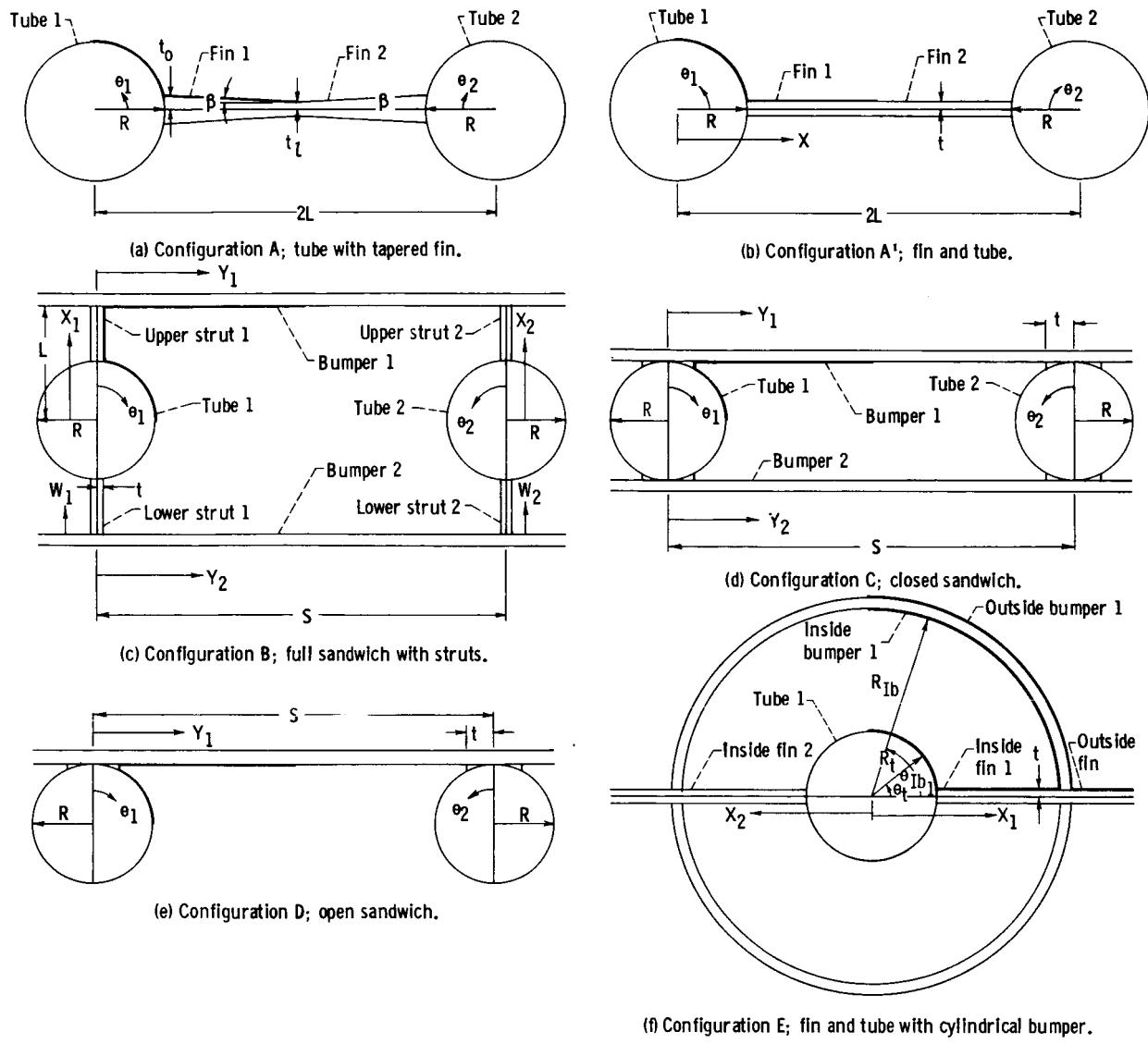


Figure 8. - Configurations.

heat-transfer calculations in fin and tube radiator configurations that are assumed to be longitudinally isothermal. Proposed direct condensing fin-tube radiators of Rankine cycle powerplants are longitudinally isothermal, and the view factors of this report can be used.

Proposed sensible heat radiators such as gas radiators of Brayton cycle powerplants and liquid radiators of Rankine cycle powerplants will not be longitudinally isothermal, and therefore the view factors presented herein for isothermal strips would not seem to be applicable to calculations in such radiators. In reference 9, however, it is shown that a radiator of this type can be divided into longitudinal segments that can reasonably be assumed to be longitudinally isothermal. Further, it was shown that the interchange from one segment to another can be neglected. For example, in figure 9, the interchange between the strip on the tube in segment 2 and the strips on the fin in seg-

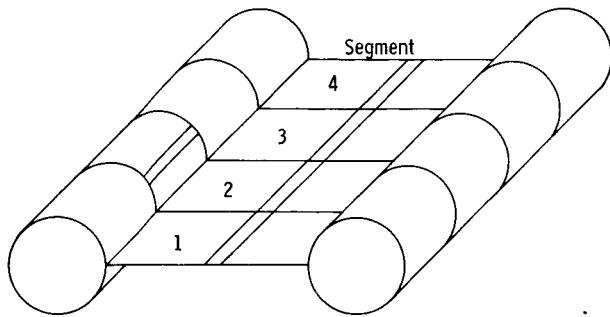


Figure 9. - Division of radiator into longitudinally isothermal segments.

how widely this conclusion could be applied to the other configurations. If the longitudinal segments could each be assumed to be isothermal and of constant radiosity in the Z-direction, however, the view-factor ratio (fig. 7, p. 10) could be used in the following manner as a measure of the applicability of the quasi-two-dimensional treatment. Divide the radiator into segments of length Z ; determine the largest value of r between two elements in a segment that see each other (this can be done simply by scaling a cross section sketch); compute Z/r and read the view-factor ratio from figure 7. If the ratio is greater than, say, 0.95, then the interchange between segments can certainly be neglected; if the ratio is less than 0.95, then further investigation should be made. Figure 7 can also be used as an aid in determining what the length of each segment should be in a longitudinally nonisothermal radiator calculation by first choosing a value of the view-factor ratio and using figure 7 to find Z .

To summarize, the results presented herein are applicable to two-dimensional heat-transfer calculations of longitudinally isothermal radiators and should also be applicable to quasi-two-dimensional calculation of nonisothermal radiators.

RESULTS

In this section, the expressions for the view factors and the limits of visibility are presented for the base surfaces of all the configurations of figure 8. (Base surfaces are indicated by heavy lines in fig. 8.) The view factors from the balance of the surfaces on the same configuration can be obtained from the base surface view factors by symmetry. For example, the view factor from dS_{t_2} to dS_{f_1} is equal to that from dS_{t_1} to dS_{f_2} for the

locations of dS shown in figure 10. For this reason, view factors and limits are given only from base surfaces to any surfaces visible to the base surfaces.

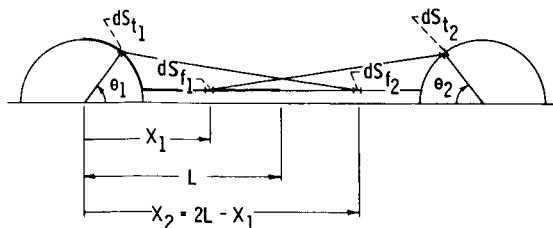


Figure 10. - Example of geometric symmetry.

ments 1, 3, and 4 can be neglected. Thus, the heat flow in each segment can be treated as if it were two dimensional, and the view factor presented herein could be used in the interchange calculations. The conclusion of reference 9 was based on the actual geometry of a practical radiator configuration of the central-fin type (as in fig. 9) with radiating surfaces that were assumed to be black. It is not immediately apparent

that this conclusion could be applied to the other configurations. If the longitudinal segments could each be assumed to be isothermal and of constant radiosity in the Z-direction, however, the view-factor ratio (fig. 7, p. 10) could be used in the following manner as a measure of the applicability of the quasi-two-dimensional treatment. Divide the radiator into segments of length Z ; determine the largest value of r between two elements in a segment that see each other (this can be done simply by scaling a cross section sketch); compute Z/r and read the view-factor ratio from figure 7. If the ratio is greater than, say, 0.95, then the interchange between segments can certainly be neglected; if the ratio is less than 0.95, then further investigation should be made. Figure 7 can also be used as an aid in determining what the length of each segment should be in a longitudinally nonisothermal radiator calculation by first choosing a value of the view-factor ratio and using figure 7 to find Z .

To summarize, the results presented herein are applicable to two-dimensional heat-transfer calculations of longitudinally isothermal radiators and should also be applicable to quasi-two-dimensional calculation of nonisothermal radiators.

RESULTS

In this section, the expressions for the view factors and the limits of visibility are presented for the base surfaces of all the configurations of figure 8. (Base surfaces are indicated by heavy lines in fig. 8.) The view factors from the balance of the surfaces on the same configuration can be obtained from the base surface view factors by symmetry. For example, the view factor from dS_{t_2} to dS_{f_1} is equal to that from dS_{t_1} to dS_{f_2} for the locations of dS shown in figure 10. For this reason, view factors and limits are given only from base surfaces to any surfaces visible to the base surfaces.

The format of presentation is as follows: First, the general formulas for view factors $dF_{dS_1-dS_2}$ and $dF_{dS_1-dS_2}(\infty)$ are given in terms of quantities a and r and their differentials. Then each

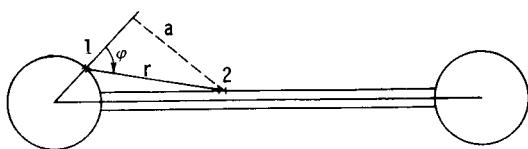


Figure 11. - View factor quantities a and r .

a , da , r , and dr for the view factor are formulated where r is the distance between differential strips and a is the projection of r onto the plane perpendicular to the normal to strip 1 (see fig. 11). The limits of visibility are then given for each case in terms of the coordinates of the configuration. For purposes of clarity, a table format is used.

In each case, the first step in formulating the limits of visibility is to determine the critical values of the independent variable. A critical value is a value of the independent variable for which the equation for a limit changes because of the geometry of the configuration (e.g., in configuration B (fig. 8(c)), tubes partially block the view of bumper 2 from some locations on bumper 1). In some cases there are no critical values, and in others there may be either one, two, or three. If there is only one critical value, a subscript c is used; however, when there are two or three, the subscripts lc , mc , and uc are used, meaning lower, middle, and upper critical, respectively. The equations for the limits of visibility are derived subject to the limitations imposed by these critical values.

The range of the independent variable is divided up by the critical values of the independent variable, if any exist. In each table, numbers are given for the upper and lower limits of visibility for the different ranges. These numbers refer to items in the list below each table.

Extra restrictions are sometimes placed on the limits in certain cases when some dimension of the cross section is unusually large or small. Methods to check on this possibility are included in the cases where this problem could arise. Because of the coordinate system chosen, in some cases the upper limit is an angle greater than 90° . In these cases, absolute value signs are used in order to avoid concern about determining the quadrant in which the angle lies.

General Formulas

The formula for the view factor for strips of infinite length is

$$dF_{dS_1-dS_2}(\infty) = \frac{1}{2} d(\sin \varphi)$$

while the formula for the view factor for strips of finite length is

$$dF_{dS_1-dS_2} = \frac{1}{\pi} d(\sin \varphi) \tan^{-1}\left(\frac{Z}{r}\right)$$

where

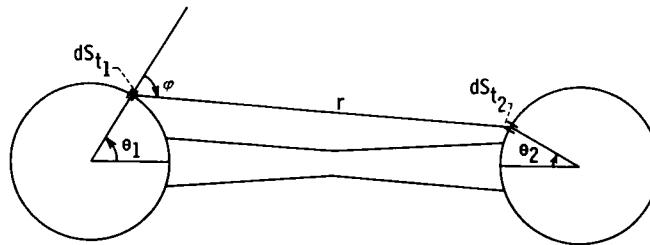
$$\sin \phi = \frac{a}{r}$$

and

$$d(\sin \phi) = \left| \frac{r da - a dr}{r^2} \right|$$

These quantities a , da , r , and dr are formulated for each case. The strip length Z will be known by the user. In some cases, the formula given for $d(\sin \phi)$ may result in a negative number; therefore, the absolute value is used to avoid concern over the sign.

Case Al: Tube 1 to Tube 2



View-factor quantities

$$a = 2L \sin \theta_1 - R \sin(\theta_1 + \theta_2)$$

$$da = -R \cos(\theta_1 + \theta_2) d\theta_2$$

$$r = \left\{ 2R^2 [1 + \cos(\theta_1 + \theta_2)] + 4L [L - R(\cos \theta_1 + \cos \theta_2)] \right\}^{1/2}$$

$$dr = \frac{2LR \sin \theta_2 - R^2 \sin(\theta_1 + \theta_2)}{r} d\theta_2$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1	
	$\sin^{-1}\left(\frac{t_o}{R}\right) \leq \theta_1 \leq (\theta_1)_c$	$(\theta_1)_c < \theta_1 \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(\theta_2)_l$	2	3
$(\theta_2)_u$	4	4

1. $(\theta_1)_c$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where fin 2 meets tube 2

$$(\theta_1)_c = \sin^{-1} \left[\frac{Rt_o + 2 \left(2L - \sqrt{R^2 - t_o^2} \right) \sqrt{L(L - \sqrt{R^2 - t_o^2})}}{4L(L - \sqrt{R^2 - t_o^2}) + R^2} \right]$$

2. The lower limit is the minimum possible value of θ_2 , that is, the point where fin 2 meets tube 2

$$(\theta_2)_l = \sin^{-1} \left(\frac{t_o}{R} \right)$$

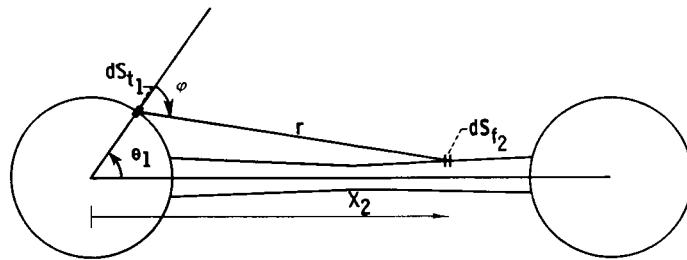
3. The lower limit is determined by a tangent to tube 1

$$(\theta_2)_l = \frac{\pi}{2} - \theta_1 - \sin^{-1} \left(\frac{2L \cos \theta_1 - R}{R} \right)$$

4. The upper limit is always determined by a tangent to tube 2

$$(\theta_2)_u = \sin^{-1} \left[\frac{R^2 \sin \theta_1 + 2(2L - R \cos \theta_1) \sqrt{L(L - R \cos \theta_1)}}{4L(L - R \cos \theta_1) + R^2} \right]$$

Case A2: Tube 1 to Fin 2



View-factor quantities

$$a = x_2 \sin \theta_1 - [(x_2 - L) \tan \beta + t_l] \cos \theta_1$$

$$da = (\sin \theta_1 - \cos \theta_1 \tan \beta) dx_2$$

$$r = [x_2^2 + R^2 + m^2 - 2R(x_2 \cos \theta_1 + m \sin \theta_1)]^{1/2}$$

$$\text{where } m = (x_2 - L) \tan \beta + t_l$$

$$dr = \frac{x_2 - R \cos \theta_1 - (R \sin \theta_1 - t_l) \tan \beta + (x_2 - L) \tan^2 \beta}{r} dx_2$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1		
	$\sin^{-1}\left(\frac{t_o}{R}\right) \leq \theta_1 \leq (\theta_1)_{lc}$	$(\theta_1)_{lc} < \theta_1 < (\theta_1)_{uc}$	$(\theta_1)_{uc} \leq \theta_1 \leq \frac{\pi}{2}$
Item numbers for limit equations			
$(x_2)_l$	2	4	6
$(x_2)_u$	5	5	6

1. $(\theta_1)_{lc}$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where fin 1 and fin 2 meet

$$(\theta_1)_{lc} = \sin^{-1} \left(\frac{Rt_l + L \sqrt{L^2 + t_l^2 - R^2}}{L^2 + t_l^2} \right)$$

2. The lower limit is the minimum possible value of x_2 , that is, the point where fin 1 meets fin 2

$$(x_2)_l = L$$

3. $(\theta_1)_{uc}$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where fin 2 meets tube 2; at values of θ_1 greater than this, tube 1 occludes the view of fin 2

$$(\theta_1)_{uc} = \sin^{-1} \left[\frac{Rt_o + 2(2L - \sqrt{R^2 - t_o^2}) \sqrt{L^2 - L(R^2 - t_o^2)^{1/2}}}{4L^2 - 4L \sqrt{R^2 - t_o^2} + R^2} \right]$$

4. The lower limit is determined by a tangent to tube 1

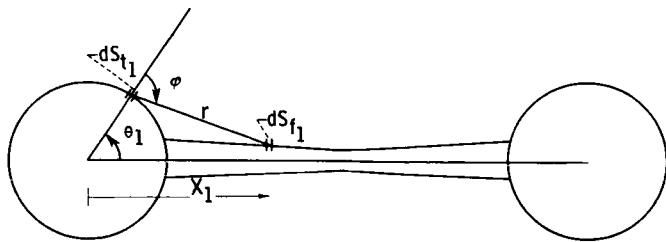
$$(x_2)_l = \frac{R - t_o \sin \theta_1 + \sin \theta_1 \tan \beta (2L - \sqrt{R^2 - t_o^2})}{\cos \theta_1 + \sin \theta_1 \tan \beta}$$

5. The upper limit is the maximum possible value of x_2 , that is, the point where fin 2 meets tube 2

$$(x_2)_u = 2L - \sqrt{R^2 - t_o^2}$$

6. Fin 2 is not visible from tube 1 for this range of θ_1 .

Case A3: Tube 1 to Fin 1



View-factor quantities

$$a = X_1 \sin \theta_1 - [(L - X_1) \tan \beta + t_l] \cos \theta_1$$

$$da = (\sin \theta_1 + \cos \theta_1 \tan \beta) dX_1$$

$$r = [X_1^2 + R^2 + m^2 - 2R(X_1 \cos \theta_1 + m \sin \theta_1)]^{1/2}$$

$$\text{where } m = (L - X_1) \tan \beta + t_l$$

$$\frac{dr}{dx_1} = \frac{X_1 - R \cos \theta_1 + (R \sin \theta_1 - t_l) \tan \beta + (X_1 - L) \tan^2 \beta}{r} dx_1$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1	
	$\sin^{-1}\left(\frac{t_o}{R}\right) \leq \theta_1 < (\theta_1)_c$	$(\theta_1)_c \leq \theta_1 \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(x_1)_l$	2	4
$(x_1)_u$	3	4

1. $(\theta_1)_c$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where fin 1 meets fin 2; at values of θ_1 greater than this, tube 1 occludes the view of fin 1

$$(\theta_1)_c = \sin^{-1}\left(\frac{Rt_l + L \sqrt{L^2 + t_l^2 - R^2}}{L^2 + t_l^2}\right)$$

2. The lower limit is determined by a tangent to tube 1

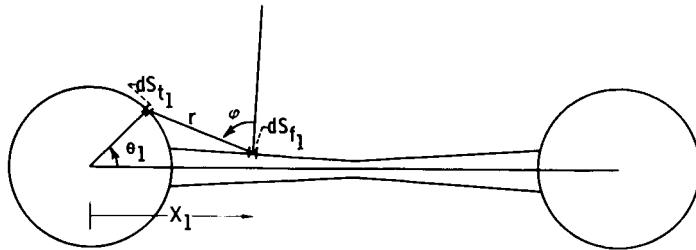
$$(x_1)_l = \frac{R - (t_o + \sqrt{R^2 - t_o^2} \tan \beta) \sin \theta_1}{\cos \theta_1 - \sin \theta_1 \tan \beta}$$

3. The upper limit is the maximum possible value of X_1 , that is, the point where fin 1 meets fin 2

$$(X_1)_u = L$$

4. Fin 1 is not visible from tube 1 for this range of θ_1 .

Case A4: Fin 1 to Tube 1



View-factor quantities

This case is related to case A3 by

$$dS_{f1} \frac{dF_{f1-t1}}{dX_1} = dS_{t1} \frac{dF_{t1-f1}}{dX_1}$$

$$dF_{f1-t1} = \frac{R}{dX_1} \frac{d\theta_1}{(cos \beta)} dF_{t1-f1}$$

or

$$a = \cos \beta [X_1 - R \cos \theta_1 + R \sin \theta_1 \tan \beta - t_l \tan \beta - (L - X_1) \tan^2 \beta]$$

$$da = \cos \beta (R \sin \theta_1 + R \tan \beta \cos \theta_1) d\theta_1$$

$$r = \left\{ (X_1 - R \cos \theta_1)^2 + [R \sin \theta_1 - t_l - (L - X_1) \tan \beta]^2 \right\}^{1/2}$$

$$\frac{dr}{d\theta_1} = \frac{X_1 R \sin \theta_1 - R \cos \theta_1 [t_l + (L - X_1) \tan \beta]}{r} d\theta_1$$

Limits of visibility

Limit of visibility	Range of independent variable X_1
	$\sqrt{R^2 - t_o^2} \leq X_1 \leq L$
	Item numbers for limit equations
$(\theta_1)_l$	1
$(\theta_1)_u$	2

1. The lower limit is always the minimum possible value of θ_1 , that is, the point where fin 1 meets tube 1

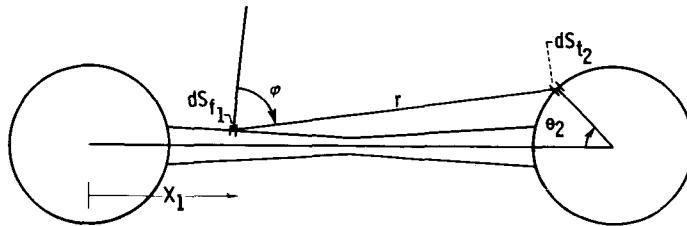
$$(\theta_1)_l = \sin^{-1}\left(\frac{t_o}{R}\right)$$

2. The upper limit is always determined by a tangent to tube 1

$$(\theta_1)_u = \sin^{-1}\left(\frac{t_R + x_1 \sqrt{x_1^2 + t^2 - R^2}}{x_1^2 + t^2}\right)$$

where $t = t_o - (x_1 - \sqrt{R^2 - t_o^2}) \tan \beta$

Case A5: Fin 1 to Tube 2



View-factor quantities

$$a = \cos \beta \{ 2L - x_1 - R \cos \theta_2 - \tan \beta [R \sin \theta_2 - t_l - (L - x_1) \tan \beta] \}$$

$$da = \cos \beta (R \sin \theta_2 - R \tan \beta \cos \theta_2) d\theta_2$$

$$r = [x_1^2 + R^2 + m^2 + 4L(L - x_1 - R \cos \theta_2) + 2R(x_1 \cos \theta_2 - m \sin \theta_2)]^{1/2}$$

where $m = \tan \beta (L - x_1) + t_l$

$$dr = \frac{R \sin \theta_2 (2L - x_1) - R \cos \theta_2 [\tan \beta (L - x_1) + t_l]}{r} d\theta_2$$

Limits of visibility

Limit of visibility	Range of independent variable x_1
	$\sqrt{R^2 - t_o^2} \leq x_1 \leq L$
	Item numbers for limit equations
$(\theta_2)_l$	1
$(\theta_2)_u$	2

1. The lower limit is always the minimum possible value of θ_2 , that is, the point where fin 2 meets tube 2

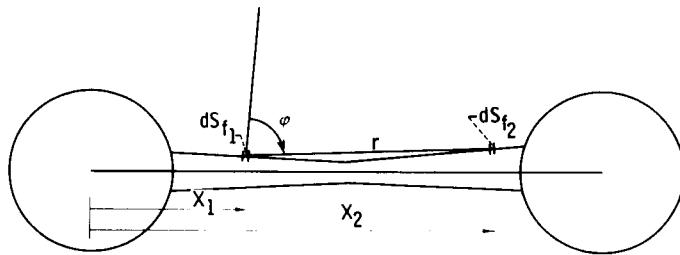
$$(\theta_2)_l = \sin^{-1}\left(\frac{t_o}{R}\right)$$

2. The upper limit is always determined by a tangent to tube 2

$$(\theta_2)_u = \sin^{-1}\left[\frac{tR + (2L - x_1)\sqrt{(2L - x_1)^2 + t^2 - R^2}}{(2L - x_1)^2 + t^2}\right]$$

$$\text{where } t = t_o - \tan \beta \left(x_1 - \sqrt{R^2 - t_o^2} \right)$$

Case A6: Fin 1 to Fin 2



View-factor quantities

$$a = \frac{(x_2 - x_1) - 2 \sin^2 \beta (x_2 - L)}{\cos \beta}$$

$$da = \frac{\cos 2\beta}{\cos \beta} dx_2$$

$$r = \frac{\left[(x_2 - x_1)^2 - 4 \sin^2 \beta (L - x_1)(x_2 - L)\right]^{1/2}}{\cos \beta}$$

$$dr = \frac{(x_2 - x_1) - 2 \sin^2 \beta (L - x_1)}{r} dx_2$$

Limits of visibility

Limit of visibility	Range of independent variable X_1
	$\sqrt{R^2 - t_o^2} \leq X_1 \leq L$
Item numbers for limit equations	
$(X_2)_l$	1
$(X_2)_u$	2

1. The lower limit is always the minimum possible value of X_2 , that is, the point where fin 1 meets fin 2

$$(X_2)_l = L$$

2. The upper limit is always the maximum possible value of X_2 , that is, the point where fin 2 meets tube 2

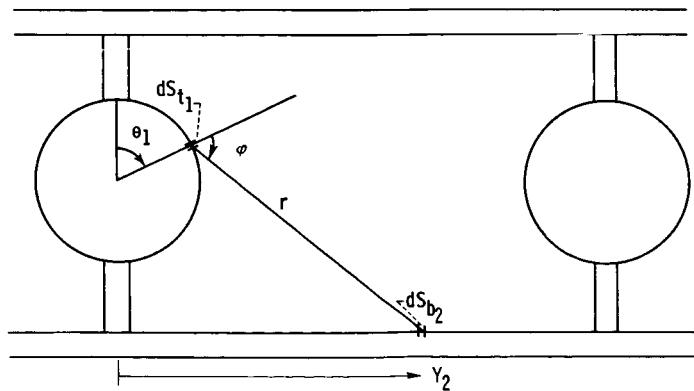
$$(X_2)_u = 2L - \sqrt{R^2 - t_o^2}$$

Configuration A': All Cases



This configuration is a special case of configuration A with a rectangular fin instead of a tapered fin. The view factors and limits of visibility can be obtained from those for configuration A by setting $\beta = 0$ and $t_o = t_l = t$.

Case B1: Tube 1 to Bumper 2



View-factor quantities

$$a = Y_2 \cos \theta_1 + L \sin \theta_1$$

$$da = \cos \theta_1 dY_2$$

$$r = \left[Y_2^2 + L^2 + R^2 + 2R(L \cos \theta_1 - Y_2 \sin \theta_1) \right]^{1/2}$$

$$dr = \frac{Y_2 - R \sin \theta_1}{r} dY_2$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1	
	$\sin^{-1}\left(\frac{t}{R}\right) \leq \theta_1 \leq (\theta_1)_c$	$(\theta_1)_c < \theta_1 \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(Y_2)_l$	1	3
$(Y_2)_u$	1	4

1. Bumper 2 is not visible from tube 1 for this range of θ_1 .

2. $(\theta_1)_c$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where lower strut 2 meets bumper 2; at values of θ_1 less than this, tube 1 occludes the view of bumper 2

$$(\theta_1)_c = \sin^{-1} \left[\frac{R(S - t) + L \sqrt{L^2 - R^2 + (S - t)^2}}{L^2 + (S - t)^2} \right]$$

3. The lower limit is determined by a tangent to tube 1

$$(Y_2)_l = \frac{L \cos \theta_1 + R}{\sin \theta_1}$$

4. The upper limit is the maximum possible value of Y_2 , that is, the point where lower strut 2 meets bumper 2

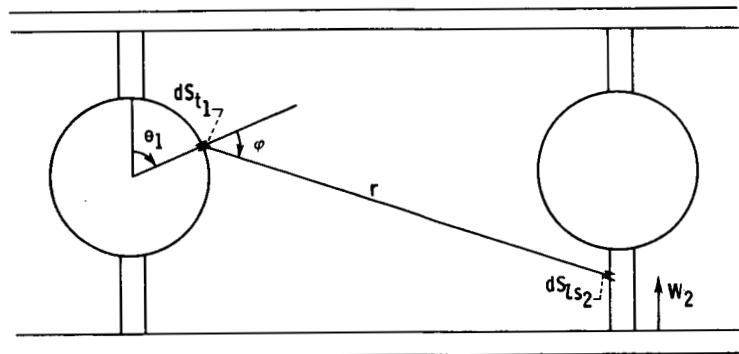
$$(Y_2)_u = S - t$$

It is possible that in some cases the dimensions of the configuration could be such that tube 2 could interfere with the upper limit of visibility. In order to determine if such is the case, compute the following value of $(Y_2)_c$:

$$(Y_2)_c = R \sin \theta_1 + (L + R \cos \theta_1) \frac{(S - R \sin \theta_1) \sqrt{S^2 - 2RS \sin \theta_1} - R^2 \cos \theta_1}{R(S - R \sin \theta_1) + R \cos \theta_1 \sqrt{S^2 - 2RS \sin \theta_1}}$$

If $(Y_2)_c \geq S - t$ where $S - t$ is the maximum possible value of Y_2 , then tube 2 does not interfere with the view of bumper 2 and $(Y_2)_u$ is given by item number 3. If $(Y_2)_c < S - t$, however, then the upper limit is determined by a tangent to tube 2 and $(Y_2)_u = (Y_2)_c$.

Case B2: Tube 1 to Lower Strut 2



View-factor quantities

$$a = (L - w_2) \sin \theta_1 + (S - t) \cos \theta_1$$

$$da = -\sin \theta_1 dw_2$$

$$r = \left[(S - t)^2 + (L - w_2)^2 + R^2 - 2R(S - t) \sin \theta_1 + 2R(L - w_2) \cos \theta_1 \right]^{1/2}$$

$$dr = \frac{w_2 - L - R \cos \theta_1}{r} dw_2$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1		
	$\sin^{-1}\left(\frac{t}{R}\right) \leq \theta_1 \leq (\theta_1)_{uc}$	$(\theta_1)_{uc} < \theta_1 < (\theta_1)_{lc}$	$(\theta_1)_{lc} \leq \theta_1 \leq \frac{\pi}{2}$
Item numbers for limit equations			
$(w_2)_l$	1	5	6
$(w_2)_u$	1	3	3

1. Lower strut 2 is not visible from tube 1 for this range of θ_1 .
2. $(\theta_1)_{uc}$ is the value of θ_1 such that a tangent to tube 1 at this point will also be tangent to tube 2; at values of θ_1 less than this, tube 1 occludes the view of lower strut 2

$$(\theta_1)_{uc} = \sin^{-1}\left(\frac{2R}{S}\right)$$

3. The upper limit is determined by a tangent to tube 2

$$(W_2)_u = L - \left[\frac{R(S^2 + R^2 - 2SR \sin \theta_1) - tR(S - R \sin \theta_1 + \sqrt{S^2 - 2SR \sin \theta_1 \cos \theta_1})}{(S - R \sin \theta_1) \sqrt{S^2 - 2SR \sin \theta_1 \cos \theta_1}} \right]$$

4. $(\theta_1)_{lc}$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where lower strut 2 meets bumper 2

$$(\theta_1)_{lc} = \sin^{-1}\left[\frac{R(S - t) + L \sqrt{L^2 - R^2 + (S - t)^2}}{L^2 + (S - t)^2}\right]$$

5. The lower limit is determined by a tangent to tube 1

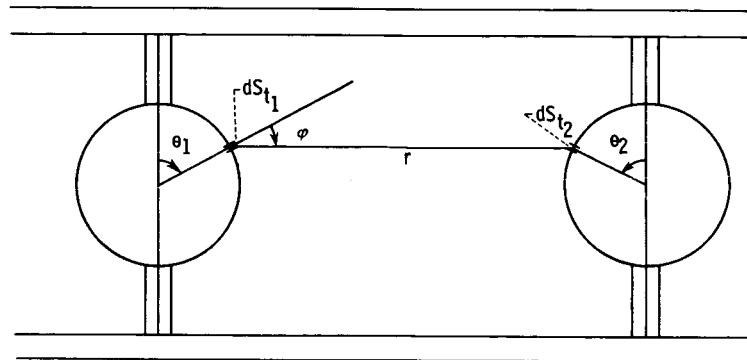
$$(W_2)_l = \frac{L \cos \theta_1 + R - (S - t) \sin \theta_1}{\cos \theta_1}$$

It is possible that in some cases the dimensions of the configuration could be such that tube 2 could completely occlude the view of lower strut 2 for some values of θ_1 . In order to determine if such is the case, compute the value of $(W_2)_u$ as given by item number 2 and call it $(W_2)_c$. If $(W_2)_c \leq 0$ where 0 is the minimum possible value of W_2 , then tube 2 occludes the view of lower strut 2 and $(W_2)_l = 0$ and $(W_2)_u = 0$. If $(W_2)_c > 0$, however, then lower strut 2 is not occluded by tube 2 and the limits are those given in the table.

6. The lower limit is the minimum possible value of W_2 , that is, the point where bumper 2 meets lower strut 2

$$(W_2)_l = 0$$

Case B3: Tube 1 to Tube 2



View-factor quantities

$$a = S \cos \theta_1 - R \sin(\theta_1 + \theta_2)$$

$$da = -R \cos(\theta_1 + \theta_2) d\theta_2$$

$$r = \left\{ S^2 - 2RS(\sin \theta_1 + \sin \theta_2) + 2R^2 [1 - \cos(\theta_1 + \theta_2)] \right\}^{1/2}$$

$$dr = \frac{R [R \sin(\theta_1 + \theta_2) - S \cos \theta_2]}{r} d\theta_2$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1	
	$\sin^{-1}\left(\frac{t}{R}\right) \leq \theta_1 \leq (\theta_1)_{lc}$	$(\theta_1)_{lc} < \theta_1 \leq \frac{\pi}{2}$
	Item numbers for limit equations	
$(\theta_2)_l$	2	3

Limit of visibility	Range of independent variable θ_1			
	$\sin^{-1}\left(\frac{t}{R}\right) \leq \theta_1 < (\theta_1)_{mc}$	$\theta_1 = (\theta_1)_{mc}$	$(\theta_1)_{mc} < \theta_1 < (\theta_1)_{uc}$	$(\theta_1)_{uc} \leq \theta_1 \leq \frac{\pi}{2}$
	Item numbers for limit equations			
$(\theta_2)_u$	6	7	8	9

1. $(\theta_1)_{lc}$ is the value of θ_1 for which the following equation will hold true; it is the value of θ_1 such that a tangent to tube 2 from this point will hit the point where upper strut 2 hits tube 2.

$$R \left[(R^2 - 2St) \sin \theta_1 + R \sqrt{S^2 - 2RS \sin \theta_1 \cos \theta_1} \right] = SR^2 - t(S^2 - R^2)$$

2. The lower limit is the minimum possible value of θ_2 , that is, the point where upper strut 2 meets tube 2

$$(\theta_2)_l = \sin^{-1}\left(\frac{t}{R}\right)$$

3. The lower limit is determined by a tangent to tube 2

$$(\theta_2)_l = \sin^{-1}\left[\frac{R(S - R \sin \theta_1) - R \cos \theta_1 \sqrt{S^2 - 2SR \sin \theta_1}}{S^2 + R^2 - 2SR \sin \theta_1}\right]$$

4. $(\theta_1)_{mc}$ is the value of θ_1 such that a tangent to tube 1 at this point will hit tube 2 at $\theta_2 = \pi/2$

$$(\theta_1)_{mc} = \sin^{-1}\left(\frac{R}{S - R}\right)$$

5. $(\theta_1)_{uc}$ is the value of θ_1 such that a tangent to tube 1 at this point will also be tangent to tube 2

$$(\theta_1)_{uc} = \sin^{-1}\left(\frac{2R}{S}\right)$$

6. The upper limit is determined by a tangent to tube 1 and is less than 90°

$$(\theta_2)_u = \sin^{-1}\left[\frac{R^2 S (\sin^2 \theta_1 + 2) + (R^2 - S^2) \sqrt{S(2R - S \sin \theta_1) \sin \theta_1} \cos \theta_1 - (R^3 - S^3 \sin \theta_1 + 3S^2 R) \sin \theta_1}{R(R^2 + S^2 - 2RS \sin \theta_1)}\right]$$

7. The upper limit is determined by a tangent to tube 1 and is equal to 90°

$$(\theta_2)_u = \frac{\pi}{2}$$

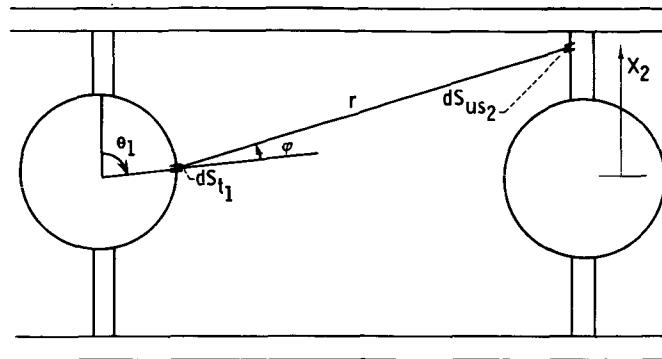
8. The upper limit is determined by a tangent to tube 1 and is greater than 90°

$$(\theta_2)_u = \pi - \left| \sin^{-1}\left[\frac{(S \sin \theta_1 - R) \sin \theta_1 - \sqrt{S(2R - S \sin \theta_1) \sin \theta_1} \cos \theta_1}{R}\right] \right|$$

9. The upper limit is determined by a tangent to tube 2

$$(\theta_2)_u = \pi - \left| \sin^{-1}\left[\frac{R \cos \theta_1 \sqrt{S^2 - 2SR \sin \theta_1} + R(S - R \sin \theta_1)}{S^2 + R^2 - 2SR \sin \theta_1}\right] \right|$$

Case B4: Tube 1 to Upper Strut 2



View-factor quantities

$$a = (S - t) \cos \theta_1 - x_2 \sin \theta_1$$

$$da = -\sin \theta_1 dx_2$$

$$r = \left[(S - t)^2 + x_2^2 + R^2 - 2R(S - t)\sin \theta_1 - 2Rx_2 \cos \theta_1 \right]^{1/2}$$

$$dr = \frac{x_2 - R \cos \theta_1}{r} dx_2$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1	
	$\sin^{-1}\left(\frac{t}{R}\right) \leq \theta_1 \leq (\theta_1)_c$	$(\theta_1)_c < \theta_1 \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(x_2)_l$	2	3
$(x_2)_u$	4	4

1. $(\theta_1)_c$ is the value of θ_1 for which the following equation will hold true; it is the value of θ_1 such that a tangent to tube 2 from this point will hit the point where upper strut 2 hits tube 2

$$\sqrt{R^2 - t^2} \left[R^2 \cos \theta_1 + (S - R \sin \theta_1) \sqrt{S^2 - 2SR \sin \theta_1} \right] =$$

$$R(S^2 + R^2 - 2SR \sin \theta_1) - Rt \left[S - R \sin \theta_1 - \left(\sqrt{S^2 - 2SR \sin \theta_1} \right) \cos \theta_1 \right]$$

2. The lower limit is the minimum possible value of X_2 , that is, the point where upper strut 2 meets tube 2

$$(X_2)_l = \sqrt{R^2 - t^2}$$

3. The lower limit is determined by a tangent to tube 2

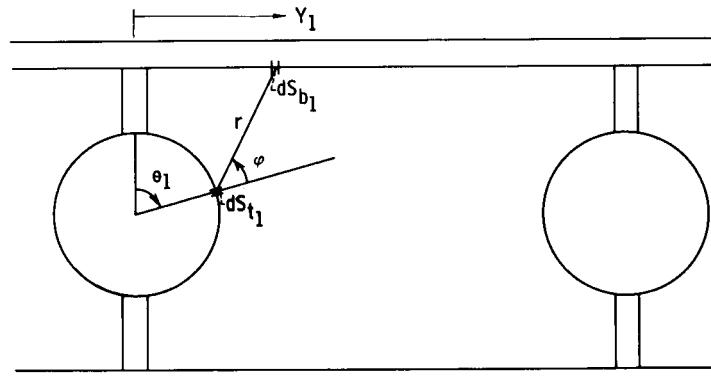
$$(X_2)_l = \frac{R(S^2 + R^2 - 2SR \sin \theta_1) - Rt(S - R \sin \theta_1 - \cos \theta_1 \sqrt{S^2 - 2SR \sin \theta_1})}{(S - R \sin \theta_1) \sqrt{S^2 - 2SR \sin \theta_1} + R^2 \cos \theta_1}$$

4. The upper limit is the maximum possible value of X_2 , that is, the point where upper strut 2 meets bumper 1

$$(X_2)_u = L$$

It is possible that in some cases the dimensions of the configuration could be such that tube 2 could completely occlude the view of upper strut 2 for some values of θ_1 . In order to determine if such is the case, compute the value of $(X_2)_l$ as given by equation (3) and call it $(X_2)_c$. If $(X_2)_c \geq L$ where L is the maximum possible value of X_2 , then tube 2 occludes the view of upper strut 2 and $(X_2)_l = 0$ and $(X_2)_u = 0$. If $(X_2)_c < L$, however, then upper strut 2 is not occluded by tube 2 and the upper limit is given by item number 4.

Case B5: Tube 1 to Bumper 1



View-factor quantities

$$a = L \sin \theta_1 - Y_1 \cos \theta_1$$

$$da = -\cos \theta_1 dy_1$$

$$r = [Y_1^2 + L^2 + R^2 - 2R(L \cos \theta_1 + Y_1 \sin \theta_1)]^{1/2}$$

$$dr = \frac{Y_1 - R \sin \theta_1}{r} dy_1$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1	
	$\sin^{-1}\left(\frac{t}{R}\right) \leq \theta_1 \leq (\theta_1)_c$	$(\theta_1)_c < \theta_1 \leq \frac{\pi}{2}$
	Item numbers for limit equations	
$(Y_1)_l$	2	3
$(Y_1)_u$	4	4

1. $(\theta_1)_c$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where upper strut 1 meets bumper 1

$$(\theta_1)_c = \sin^{-1} \left(\frac{Rt + L \sqrt{L^2 + t^2 - R^2}}{L^2 + t^2} \right)$$

2. The lower limit is the minimum possible value of Y_1 , that is, the point where upper strut 1 meets bumper 1

$$(Y_1)_l = t$$

3. The lower limit is determined by a tangent to tube 1

$$(Y_1)_l = \frac{R - L \cos \theta_1}{\sin \theta_1}$$

4. The upper limit is the maximum possible value of Y_1 , that is, the point where upper strut 2 meets bumper 1

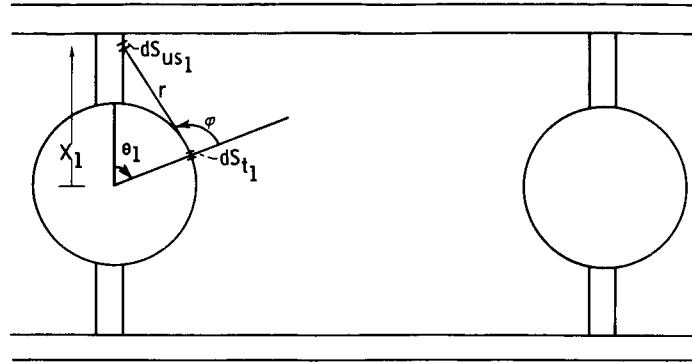
$$(Y_1)_u = S - t$$

It is possible that in some cases the dimensions of the configuration could be such that tube 2 could interfere with the upper limit of visibility. In order to determine if such is the case, compute the following value of $(Y_1)_c$:

$$(Y_1)_c = R \sin \theta_1 + (L - R \cos \theta_1) \frac{R^2 \cos \theta_1 - (R \sin \theta_1 - S) \sqrt{S^2 - 2RS \sin \theta_1}}{RS - R^2 \sin \theta_1 - R \cos \theta_1 \sqrt{S^2 - 2RS \sin \theta_1}}$$

If $(Y_1)_c \geq S - t$ where $S - t$ is the maximum possible value of Y_1 , then tube 2 does not interfere with the view of bumper 1 and $(Y_1)_u$ is given by item number 4. If $(Y_1)_c < S - t$, however, then the upper limit is determined by a tangent to tube 2 and $(Y_1)_u = (Y_1)_c$.

Case B6: Tube 1 to Upper Strut 1



View-factor quantities

This case can be obtained from Case A3 by making the following substitutions:

$$\beta = 0 \quad (t_l)_{A3} = (t)_{B6}$$

where the subscripts denote the case to which the symbol belongs. Then

$$a = X_1 \sin \theta_1 - t \cos \theta_1$$

$$da = \sin \theta_1 dX_1$$

$$r = [X_1^2 + R^2 + t^2 - 2R(X_1 \cos \theta_1 + t \sin \theta_1)]^{1/2}$$

$$dr = \frac{X_1 - R \cos \theta_1}{r} dX_1$$

Limits of visibility

Limit of visibility	Range of independent variable θ_1	
	$\sin^{-1}\left(\frac{t}{R}\right) \leq \theta_1 < (\theta_1)_c$	$(\theta_1)_c \leq \theta_1 \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(X_1)_l$	2	4
$(X_1)_u$	3	4

1. $(\theta_1)_c$ is the value of θ_1 such that a tangent to tube 1 at this point will hit the point where upper strut 1 meets bumper 1

$$(\theta_1)_c = \sin^{-1} \left(\frac{Rt + L \sqrt{L^2 + t^2 - R^2}}{L^2 + t^2} \right)$$

2. The lower limit is determined by a tangent to tube 1

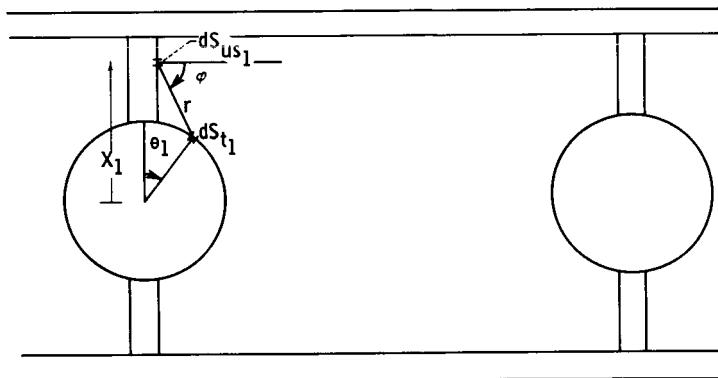
$$(x_1)_l = \frac{R - t \sin \theta_1}{\cos \theta_1}$$

3. The upper limit is the maximum possible value of x_1 , that is, the point where upper strut 1 meets bumper 1

$$(x_1)_u = L$$

4. Upper strut 1 is not visible from tube 1 for this range of θ_1 .

Case B7: Upper Strut 1 to Tube 1



View-factor quantities

This case is related to Case B6 by

$$dS_{us1} dF_{us1-t1} = dS_{t1} dF_{t1-us1}$$

$$dF_{us1-t1} = \frac{R d\theta_1}{dx_1} dF_{t1-us1}$$

or

$$a = X_1 - R \cos \theta_1$$

$$da = R \sin \theta_1 d\theta_1$$

$$r = \left[X_1^2 + R^2 + t^2 - 2R(t \sin \theta_1 + X_1 \cos \theta_1) \right]^{1/2}$$

$$dr = \frac{R(X_1 \sin \theta_1 - t \cos \theta_1)}{r} d\theta_1$$

Limits of visibility

Limit of visibility	Range of independent variable X_1
	$\sqrt{R^2 - t^2} \leq X_1 \leq L$
	Item numbers for limit equations
$(\theta_1)_l$	1
$(\theta_1)_u$	2

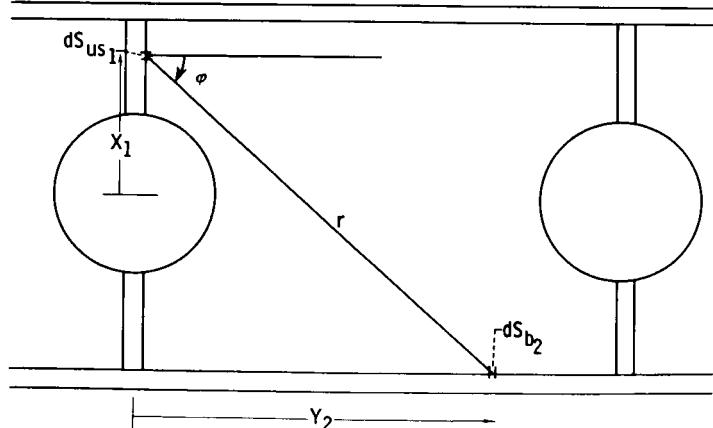
1. The lower limit is always the minimum possible value of θ_1 , that is, the point where upper strut 1 meets tube 1

$$(\theta_1)_l = \sin^{-1}\left(\frac{t}{R}\right)$$

2. The upper limit is always determined by a tangent to tube 1

$$(\theta_1)_u = \cos^{-1}\left(\frac{RX_1 + t \sqrt{X_1^2 + t^2 - R^2}}{X_1^2 + t^2}\right)$$

Case B8: Upper Strut 1 to Bumper 2



View-factor quantities

$$a = L + X_1$$

$$da = 0$$

$$r = \left[(L + X_1)^2 + (Y_2 - t)^2 \right]^{1/2}$$

$$dr = \frac{Y_2 - t}{r} dY_2$$

Limits of visibility

Limit of visibility	Range of independent variable X_1	
	$\sqrt{R^2 - t^2} \leq X_1 \leq (X_1)_c$	$(X_1)_c < X_1 \leq L$
Item numbers for limit equations		
$(Y_2)_l$	1	3
$(Y_2)_u$	1	4

1. Bumper 2 is not visible from upper strut 1 for this range of X_1 .

2. $(X_1)_c$ is the value of X_1 such that a line drawn from this point tangent to tube 1 will hit bumper 2 at the point where bumper 2 meets lower strut 2; at values of X_1 less than this, tube 1 occludes the view of bumper 2

$$(X_1)_c = -(S - 2t) \left[\frac{R(S - t) + L \sqrt{L^2 + (S - t)^2 - R^2}}{RL - (S - t) \sqrt{L^2 + (S - t)^2 - R^2}} \right] - L$$

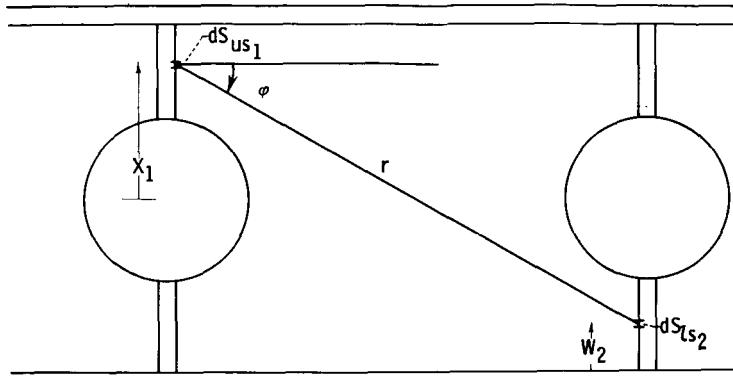
3. The lower limit is determined by a tangent to tube 1

$$(Y_2)_l = \frac{R(X_1^2 + t^2 + LX_1) - Lt \sqrt{X_1^2 + t^2 - R^2}}{Rt + X_1 \sqrt{X_1^2 + t^2 - R^2}}$$

4. The upper limit is the maximum possible value of Y_2 , that is, the point where bumper 2 meets lower strut 2

$$(Y_2)_u = S - t$$

Case B9: Upper Strut 1 to Lower Strut 2



View-factor quantities

$$a = L - w_2 + x_1$$

$$da = -dw_2$$

$$r = \left[(L + x_1 - w_2)^2 + (s - 2t)^2 \right]^{1/2}$$

$$dr = \frac{w_2 - (L + x_1)}{r} dw_2$$

Limits of visibility

Limit of visibility	Range of independent variable x_1		
	$\sqrt{R^2 - t^2} \leq x_1 \leq (x_1)_{uc}$	$(x_1)_{uc} < x_1 < (x_1)_{lc}$	$(x_1)_{lc} \leq x_1 \leq L$
	Item numbers for limit equations		
$(w_2)_l$	1	5	6
$(w_2)_u$	1	3	3

1. Lower strut 2 is not visible from upper strut 1 for this range of x_1 .

2. $(x_1)_{uc}$ is the value of x_1 such that a line drawn tangent to tube 1 from x_1 on upper strut 1 will also be tangent to tube 2; at values of x_1 less than this, tube 1 occludes the view of lower strut 2

$$(x_1)_{uc} = \frac{R(s - 2t)}{\sqrt{s^2 - 4R^2}}$$

3. The upper limit is determined by a tangent to tube 2

$$(W_2)_u = L - \left\{ \frac{R(S-t)^2 + RX_1^2 - t \left[X_1 \sqrt{(S-t)^2 + X_1^2 - R^2} + R(S-t) \right]}{(S-t) \sqrt{(S-t)^2 + X_1^2 - R^2} - RX_1} \right\}$$

4. $(X_1)_{lc}$ is the value of X_1 such that a line drawn tangent to tube 1 from X_1 on upper strut 1 will hit the point where bumper 2 meets lower strut 2

$$(X_1)_{lc} = -(S-2t) \left[\frac{R(S-t) + L \sqrt{L^2 + (S-t)^2 - R^2}}{RL - (S-t) \sqrt{L^2 + (S-t)^2 - R^2}} \right] - L$$

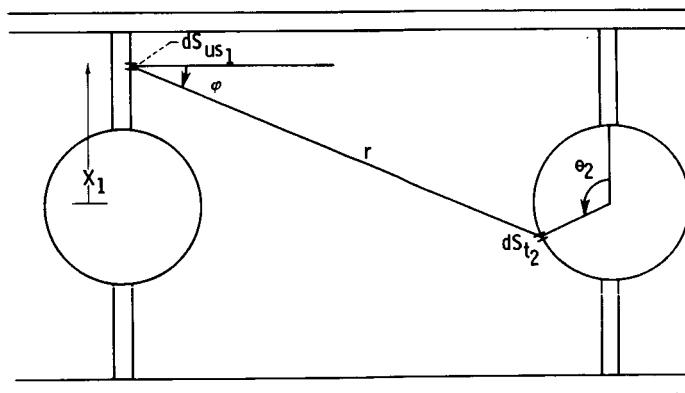
5. The lower limit is determined by a tangent to tube 1

$$(W_2)_l = L - \left\{ \frac{R[t(S-2t) - X_1^2] + X_1(S-t) \sqrt{X_1^2 + t^2 - R^2}}{X_1 R - t \sqrt{X_1^2 + t^2 - R^2}} \right\}$$

6. The lower limit is the minimum possible value of W_2 , that is, the point where lower strut 2 meets bumper 2

$$(W_2)_l = 0$$

Case B10: Upper strut 1 to Tube 2



View-factor quantities

$$a = X_1 - R \cos \theta_2$$

$$da = R \sin \theta_2 d\theta_2$$

$$r = \left[R^2 + S^2 + X_1^2 + t^2 + 2t(R \sin \theta_2 - S) - 2R(X_1 \cos \theta_2 + S \sin \theta_2) \right]^{1/2}$$

$$dr = \frac{R[X_1 \sin \theta_2 - (S - t) \cos \theta_2]}{r} d\theta_2$$

Limits of visibility

Limit of visibility	Range of independent variable X_1			
	$\sqrt{R^2 - t^2} \leq X_1 < (X_1)_{mc}$	$X_1 = (X_1)_{mc}$	$(X_1)_{mc} < X_1 < (X_1)_{uc}$	$(X_1)_{uc} \leq X_1 \leq L$
Item numbers for limit equations				
$(\theta_2)_l$	1	1	1	1
$(\theta_2)_u$	4	5	6	7

1. The lower limit is always the minimum possible value of θ_2 , that is, the point where upper strut 2 meets tube 2

$$(\theta_2)_l = \sin^{-1}\left(\frac{t}{R}\right)$$

2. $(X_1)_{mc}$ is the value of X_1 such that a tangent to tube 1 from this point will hit tube 2 at $\theta_2 = \pi/2$

$$(X_1)_{mc} = \frac{R(S - R) - tR}{\sqrt{S(S - 2R)}}$$

3. $(X_1)_{uc}$ is the value of X_1 such that a tangent to tube 1 from this point will also be tangent to tube 2

$$(X_1)_{uc} = \frac{R(S - 2t)}{\sqrt{S^2 - 4R^2}}$$

4. The upper limit is determined by a tangent to tube 1 and is less than 90°

$$(\theta_2)_u = \delta$$

5. The upper limit is determined by a tangent to tube 1 and is equal to 90°

$$(\theta_2)_u = \frac{\pi}{2}$$

6. The upper limit is determined by a tangent to tube 1 and is greater than 90°

$$(\theta_2)_u = \pi - |\delta|$$

For items 4, 5, and 6,

$$\delta = \sin^{-1} \left[\frac{(S \sin \epsilon - R) \sin \epsilon - \sqrt{S \sin \epsilon (2R - S \sin \epsilon) \cos \epsilon}}{R} \right]$$

where

$$\sin \epsilon = \frac{Rt + x_1}{\sqrt{x_1^2 + t^2 - R^2}} / \frac{x_1^2 + t^2}{x_1^2 + t^2}$$

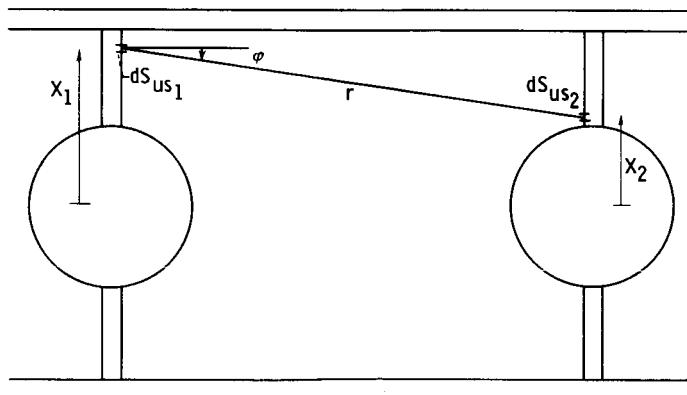
and

$$\cos \epsilon = \frac{Rx_1 - t}{\sqrt{x_1^2 + t^2 - R^2}} / \frac{x_1^2 + t^2}{x_1^2 + t^2}$$

7. The upper limit is determined by a tangent to tube 2

$$(\theta_2)_u = \pi - \left| \sin^{-1} \left[\frac{x_1 \sqrt{(S-t)^2 + x_1^2 - R^2} + R(S-t)}{(S-t)^2 + x_1^2} \right] \right|$$

Case Bll: Upper Strut 1 to Upper Strut 2



View-factor quantities

$$a = X_2 - X_1$$

$$da = dX_2$$

$$r = \left[(X_2 - X_1)^2 + (s - 2t)^2 \right]^{1/2}$$

$$dr = \frac{X_2 - X_1}{r} dx_2$$

Limits of visibility

Limit of visibility	Range of independent variable X_1
	$\sqrt{R^2 - t^2} \leq X_1 \leq L$
	Item numbers for limit equations
$(X_2)_l$	1
$(X_2)_u$	2

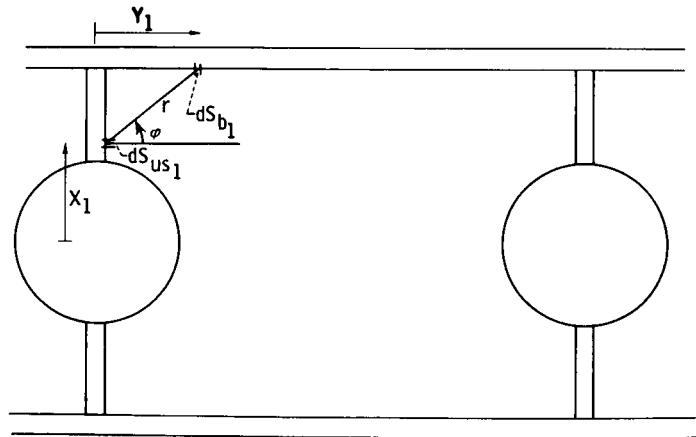
1. The lower limit is always the minimum possible value of X_2 , that is, the point where upper strut 2 meets tube 2

$$(X_2)_l = \sqrt{R^2 - t^2}$$

2. The upper limit is always the maximum possible value of X_2 , that is, the point where upper strut 2 meets bumper 1

$$(X_2)_u = L$$

Case Bl2: Upper Strut 1 to Bumper 1



View-factor quantities

$$a = L - X_1$$

$$da = 0$$

$$r = \left[(L - X_1)^2 + (Y_1 - t)^2 \right]^{1/2}$$

$$dr = \frac{Y_1 - t}{r} dY_1$$

Limits of visibility

Limit of visibility	Range of independent variable X_1
	$\sqrt{R^2 - t^2} \leq X_1 \leq L$
Item numbers for limit equations	
$(Y_1)_l$	1
$(Y_1)_u$	2

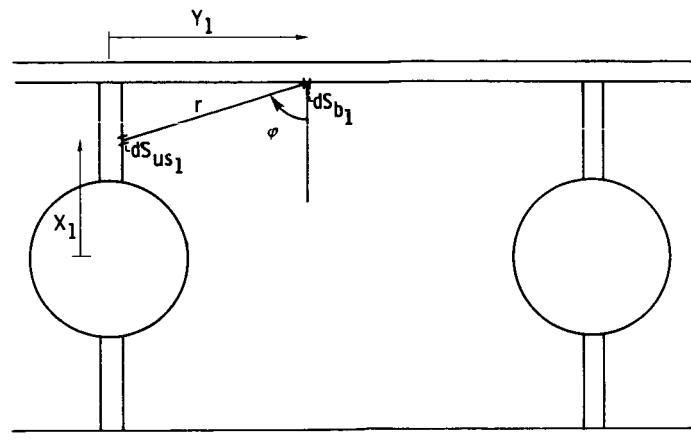
1. The lower limit is always the minimum possible value of Y_1 , that is, the point where upper strut 1 meets bumper 1

$$(Y_1)_l = t$$

2. The upper limit is always the maximum possible value of Y_1 , that is, the point where upper strut 2 meets bumper 1

$$(Y_1)_u = S - t$$

Case B13: Bumper 1 to Upper Strut 1



View-factor quantities

This case is related to Case Bl2 by

$$dS_{b_1} dF_{b_1-us_1} = dS_{us_1} dF_{us_1-b_1}$$

$$dF_{b_1-us_1} = \frac{dX_1}{dY_1} dF_{us_1-b_1}$$

or

$$a = Y_1 - t$$

$$da = 0$$

$$r = [(Y_1 - t)^2 + (L - X_1)^2]^{1/2}$$

$$dr = \frac{X_1 - L}{r} dX_1$$

Limits of visibility

Limit of visibility	Range of independent variable Y_1
	$t \leq Y_1 \leq S/2$
	Item numbers for limit equations
$(X_1)_l$	1
$(X_1)_u$	2

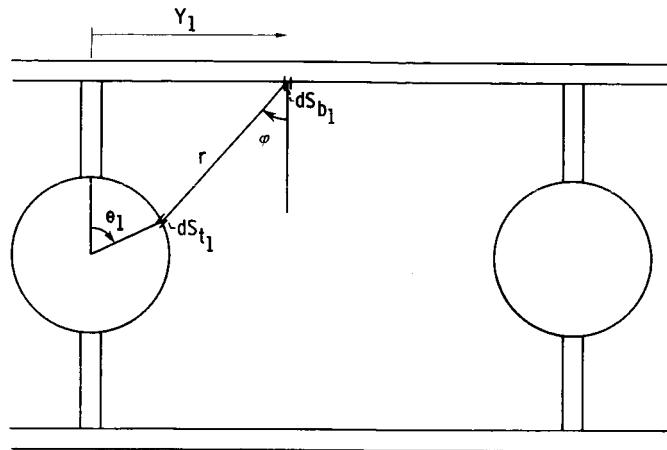
1. The lower limit is always the minimum possible value of X_1 , that is, the point where upper strut 1 meets tube 1

$$(X_1)_l = \sqrt{R^2 - t^2}$$

2. The upper limit is always the maximum possible value of X_1 , that is, the point where upper strut 1 meets bumper 1

$$(X_1)_u = L$$

Case B14: Bumper 1 to Tube 1



View-factor quantities

This case is related to Case B5 by

$$dS_{b1} \, dF_{b1-t1} = dS_{t1} \, dF_{t1-b1}$$

$$dF_{b1-t1} = \frac{R \, d\theta_1}{dY_1} \, dF_{t1-b1}$$

or

$$a = Y_1 - R \sin \theta_1$$

$$da = -R \cos \theta_1 \, d\theta_1$$

$$r = [Y_1^2 + R^2 + L^2 - 2R(Y_1 \sin \theta_1 + L \cos \theta_1)]^{1/2}$$

$$\frac{dr}{r} = \frac{R(L \sin \theta_1 - Y_1 \cos \theta_1)}{r} \, d\theta_1$$

Limits of visibility

Limit of visibility	Range of independent variable Y_1		
	$t \leq Y_1 < (Y_1)_c$	$Y_1 = (Y_1)_c$	$(Y_1)_c < Y_1 \leq s/2$
Item numbers for limit equations			
$(\theta_1)_l$	1	1	1
$(\theta_1)_u$	3	4	5

1. The lower limit is always the minimum possible value of θ_1 , that is, the point where upper strut 1 meets tube 1

$$(\theta_1)_l = \sin^{-1}\left(\frac{t}{R}\right)$$

2. $(Y_1)_c$ is the value of Y_1 such that a tangent to tube 1 from this point will hit tube 1 at $\theta_1 = 90^\circ$

$$(Y_1)_c = R$$

3. The upper limit is determined by a tangent to tube 1 and is less than 90°

$$(\theta_1)_u = \sin^{-1} \left(\frac{L \sqrt{Y_1^2 + L^2 - R^2} + RY_1}{Y_1^2 + L^2} \right)$$

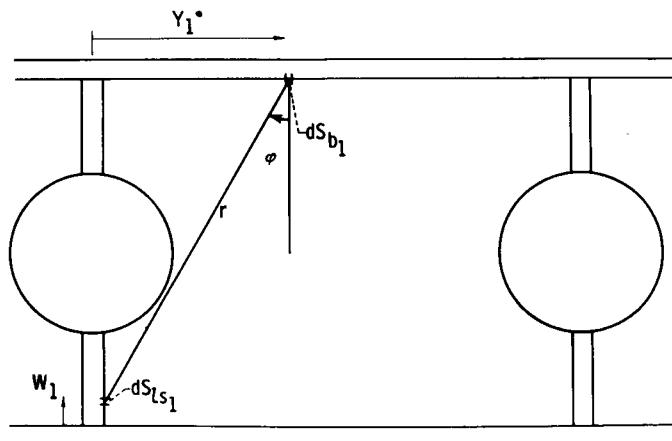
4. The upper limit is determined by a tangent to tube 1 and is equal to 90°

$$(\theta_1)_u = \frac{\pi}{2}$$

5. The upper limit is determined by a tangent to tube 1 and is greater than 90°

$$(\theta_1)_u = \pi - \left| \sin^{-1} \left(\frac{L \sqrt{Y_1^2 + L^2 - R^2} + RY_1}{Y_1^2 + L^2} \right) \right|$$

Case Bl5: Bumper 1 to Lower Strut 1



View-factor quantities

$$a = Y_1 - t$$

$$da = 0$$

$$r = \left[(2L - w_1)^2 + (Y_1 - t)^2 \right]^{1/2}$$

$$dr = \frac{w_1 - 2L}{r} dw_1$$

Limits of visibility

Limit of visibility	Range of independent variable Y_1	
	$t \leq Y_1 \leq (Y_1)_c$	$(Y_1)_c < Y_1 \leq s/2$
Item numbers for limit equations		
$(w_1)_l$	1	3
$(w_1)_u$	1	4

1. Lower strut l is not visible from bumper 1 for this range of Y_1 .

2. $(Y_1)_c$ is the value of Y_1 such that a tangent to tube l from this point will hit the point where lower strut l meets bumper 2; at values of Y_1 less than this, tube l occludes the view of lower strut l

$$(Y_1)_c = \frac{R(2L^2 + t^2) - Lt \sqrt{t^2 + L^2 - R^2}}{Rt + L \sqrt{t^2 + L^2 - R^2}}$$

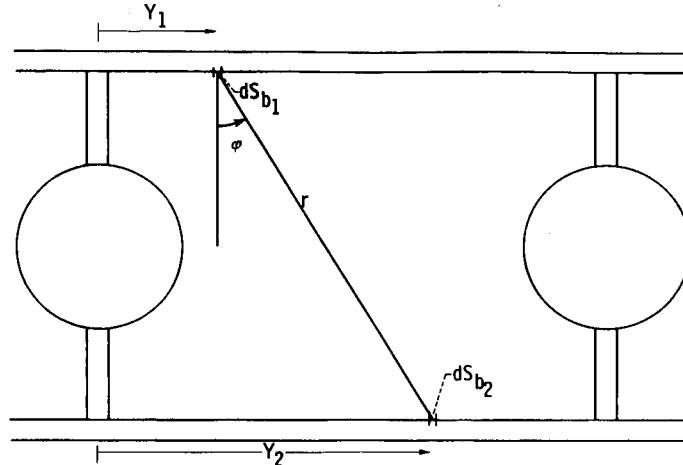
3. The lower limit is the minimum possible value of w_1 , that is, the point where lower strut l meets bumper 2

$$(w_1)_l = 0$$

4. The upper limit is determined by a tangent to tube l

$$(w_1)_u = L - \left[\frac{R(Y_1^2 - Y_1 t + L^2) - Lt \sqrt{L^2 + Y_1^2 - R^2}}{Y_1 \sqrt{L^2 + Y_1^2 - R^2} - RL} \right]$$

Case Bl6: Bumper 1 to Bumper 2



View-factor quantities

$$a = Y_2 - Y_1$$

$$da = dY_2$$

$$r = \left[(Y_2 - Y_1)^2 + 4L^2 \right]^{1/2}$$

$$dr = \frac{Y_2 - Y_1}{r} dY_2$$

Limits of visibility

Limit of visibility	Range of independent variable Y_1	
	$t \leq Y_1 < (Y_1)_{lc}$	$(Y_1)_{lc} \leq Y_1 \leq S/2$
Item numbers for limit equations		
$(Y_2)_l$	2	3

Limit of visibility	Range of independent variable Y_1	
	$t \leq Y_1 \leq (Y_1)_{uc}$	$(Y_1)_{uc} < Y_1 \leq S/2$
Item numbers for limit equations		
$(Y_2)_u$	5	6

1. $(Y_1)_{lc}$ is the value of Y_1 such that a tangent to tube 1 from this point will hit the point where lower strut 1 meets bumper 2

$$(Y_1)_{lc} = \frac{R(2L^2 + t^2) - Lt \sqrt{t^2 + L^2 - R^2}}{Rt + L \sqrt{t^2 + L^2 - R^2}}$$

2. The lower limit is determined by a tangent to tube 1

$$(Y_2)_l = \frac{R(2L^2 + Y_1^2) - LY_1 \sqrt{Y_1^2 + L^2 - R^2}}{RY_1 + L \sqrt{Y_1^2 + L^2 - R^2}}$$

3. The lower limit is the minimum possible value of Y_2 , that is, the point where lower strut 1 meets bumper 2

$$(Y_2)_l = t$$

4. $(Y_1)_{uc}$ is the value of Y_1 such that a tangent to tube 2 from this point will hit the point where lower strut 2 meets bumper 2

$$(Y_1)_{uc} = \frac{R[t(S - t) - 2L^2] + L(S + t) \sqrt{t^2 + L^2 - R^2}}{Rt + L \sqrt{t^2 + L^2 - R^2}}$$

5. The upper limit is the maximum possible value of Y_2 , that is, the point where lower strut 2 meets bumper 2

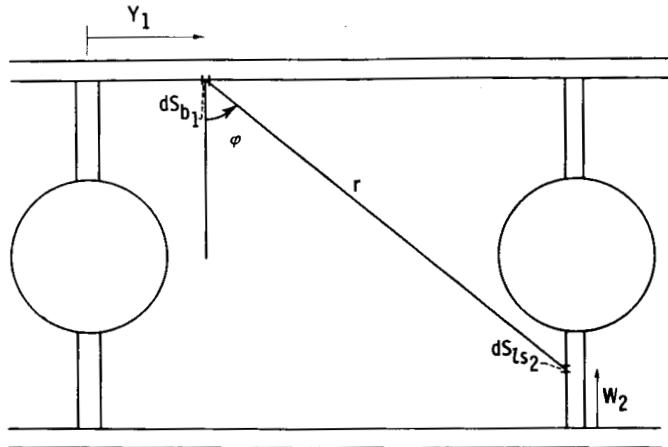
$$(Y_2)_u = S - t$$

6. The upper limit is determined by a tangent to tube 2

$$(Y_2)_u = \frac{R[Y_1(S - Y_1) - 2L^2] + L(2S - Y_1) \sqrt{L^2 + (S - Y_1)^2 - R^2}}{L \sqrt{L^2 + (S - Y_1)^2 - R^2} + R(S - Y_1)}$$

It is possible that in some cases the dimensions of the configuration could be such that tube 1 could completely occlude the view of bumper 2 for some values of Y_1 . In order to determine if such is the case, compute the value of $(Y_2)_l$ as given by item number 2 and call it $(Y_2)_c$. If $(Y_2)_c \geq S - t$ where $S - t$ is the maximum possible value of Y_2 , then tube 1 occludes the view of bumper 2 and $(Y_2)_l = 0$ and $(Y_2)_u = 0$. If $(Y_2)_c < S - t$, however, then bumper 2 is not occluded by tube 1 and the limits are those given in the table.

Case Bl7: Bumper 1 to Lower Strut 2



View-factor quantities

$$a = S - t - Y_1$$

$$da = 0$$

$$r = [(S - t - Y_1)^2 + (2L - w_2)^2]^{1/2}$$

$$\frac{dr}{dw_2} = \frac{w_2 - 2L}{r} dw_2$$

Limits of visibility

Limit of visibility	Range of independent variable Y_1	
	$t \leq Y_1 < (Y_1)_c$	$(Y_1)_c \leq Y_1 \leq S/2$
Item numbers for limit equations		
$(w_2)_l$	2	4
$(w_2)_u$	3	4

1. $(Y_1)_c$ is the value of Y_1 such that a tangent to tube 2 from this point will hit the point where lower strut 2 meets bumper 2; at values of Y_1 greater than this, tube 2 occludes the view of lower strut 2

$$(Y_1)_c = \frac{R[t(s - t) - 2L^2]}{Rt + L} + \frac{L(s + t)}{\sqrt{L^2 + t^2 - R^2}}$$

2. The lower limit is the minimum possible value of w_2 , that is, the point where lower strut 2 meets bumper 2

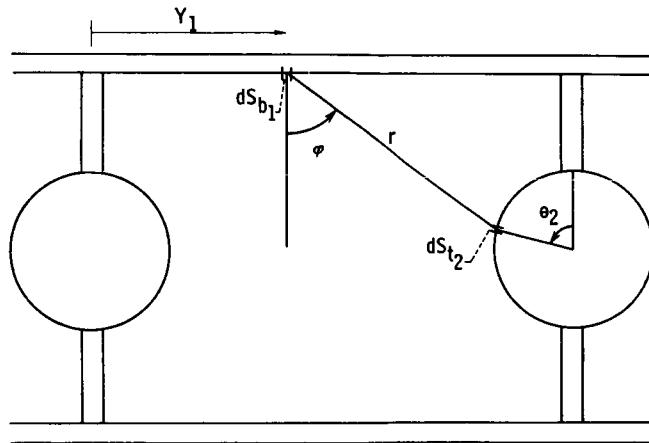
$$(w_2)_l = 0$$

3. The upper limit is determined by a tangent to tube 2

$$(w_2)_u = 2L - (S - Y_1 - t) \left[\frac{L \sqrt{L^2 + (S - Y_1)^2 - R^2} + R(S - Y_1)}{(S - Y_1) \sqrt{L^2 + (S - Y_1)^2 - R^2} - RL} \right]$$

4. Lower strut 2 is not visible from bumper 1 for this range of Y_1 .

Case B18: Bumper 1 to Tube 2



View-factor quantities

$$a = S - Y_1 - R \sin \theta_2$$

$$da = -R \cos \theta_2 d\theta_2$$

$$r = \left[(L - R \cos \theta_2)^2 + (S - Y_1 - R \sin \theta_2)^2 \right]^{1/2}$$

$$dr = \frac{(L - R \cos \theta_2)R \sin \theta_2 - R(S - Y_1 - R \sin \theta_2) \cos \theta_2}{r} d\theta_2$$

Limits of visibility

Limit of visibility	Range of independent variable Y_1			
	$t \leq Y_1 < (Y_1)_{mc}$	$Y_1 = (Y_1)_{mc}$	$(Y_1)_{mc} < Y_1 < (Y_1)_{uc}$	$(Y_1)_{uc} \leq Y_1 \leq S/2$
	Item numbers for limit equations			
$(\theta_2)_l$	1	1	1	1
$(\theta_2)_u$	4	5	6	7

1. The lower limit is always the minimum possible value of θ_2 , that is, the point where upper strut 2 meets tube 2

$$(\theta_2)_l = \sin^{-1}\left(\frac{t}{R}\right)$$

2. $(Y_1)_{uc}$ is the value of Y_1 such that a tangent to tube 1 from this point will also be tangent to tube 2

$$(Y_1)_{uc} = \frac{SR - L \sqrt{S^2 - 4R^2}}{2R}$$

3. $(Y_1)_{mc}$ is the value of Y_1 such that a tangent to tube 1 from this point will hit tube 2 at $\theta_2 = \pi/2$

$$(Y_1)_{mc} = \frac{R(S - R) - L \sqrt{S(S - 2R)}}{R}$$

4. The upper limit is determined by a tangent to tube 1 and is less than 90°

$$(\theta_2)_u = \frac{\pi}{2} - |\delta|$$

5. The upper limit is determined by a tangent to tube 1 and is equal to 90°

$$(\theta_2)_u = \frac{\pi}{2}$$

6. The upper limit is determined by a tangent to tube 1 and is greater than 90°

$$(\theta_2)_u = \frac{\pi}{2} + |\delta|$$

For items 4, 5, and 6,

$$\delta = \sin^{-1} \left\{ \frac{S \sin \omega - R}{R} \left(\frac{LR - Y_1 \sqrt{Y_1^2 + L^2 - R^2}}{Y_1^2 + L^2} \right) + \sin \omega \left[\frac{\sqrt{S \sin \omega (2R - S \sin \omega)}}{R} \right] \right\}$$

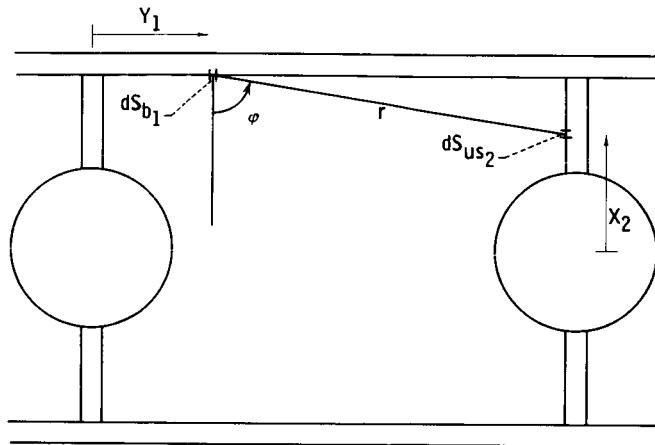
where

$$\sin \omega = \frac{Y_1 R + L \sqrt{Y_1^2 + L^2 - R^2}}{Y_1^2 + L^2}$$

7. The upper limit is determined by a tangent to tube 2

$$(\theta_2)_u = \pi - \left| \sin^{-1} \left[\frac{R(S - Y_1) + L \sqrt{L^2 + (S - Y_1)^2 - R^2}}{L^2 + (S - Y_1)^2} \right] \right|$$

Case B19: Bumper 1 to Upper Strut 2



View-factor quantities

$$a = S - Y_1 - t$$

$$da = 0$$

$$r = [(L - x_2)^2 + (S - Y_1 - t)^2]^{1/2}$$

$$dr = \frac{L - x_2}{r} dx_2$$

Limits of visibility

Limit of visibility	Range of independent variable Y_1
	$t \leq Y_1 \leq S/2$
	Item numbers for limit equations
$(x_2)_l$	1
$(x_2)_u$	2

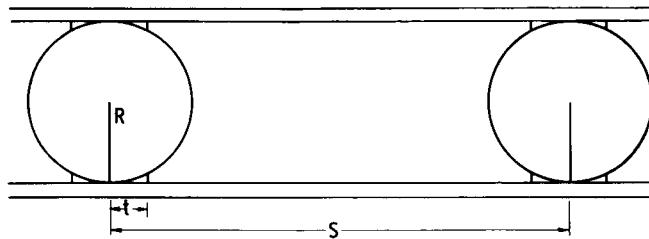
1. The lower limit is always the minimum possible value of X_2 ; that is, the point where upper strut 2 meets tube 2

$$(X_2)_l = \sqrt{R^2 - t^2}$$

2. The upper limit is always the maximum possible value of X_2 , that is, the point where upper strut 2 meets bumper 1

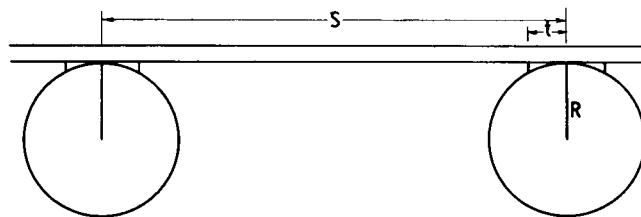
$$(X_2)_u = L$$

Configuration C: All Cases



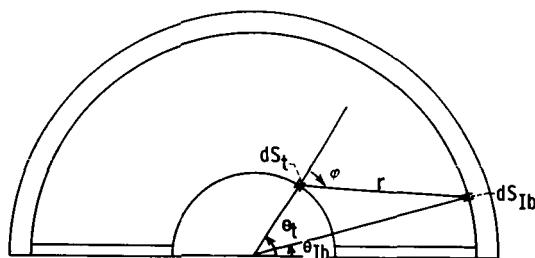
This configuration is a special case of configuration B where $L = R$. The view factors and limits of visibility for this configuration can be obtained by substitution into the corresponding equations of the cases of configuration B. Note that t may vary from zero to R . (When $t \geq R$, then the tube surface no longer contributes to the radiant interchange.)

Configuration D: All Cases



This configuration is a special case of configuration C where bumper 2 has been removed. The view factors and limits of visibility for this configuration can be obtained from the corresponding equations of the cases of configuration C that apply here.

Case El: Tube to Inside of Bumper



View-factor quantities

$$a = R_{Ib} \sin(\theta_t - \theta_{Ib})$$

$$da = -R_{Ib} \cos(\theta_t - \theta_{Ib}) d\theta_{Ib}$$

$$r = \left[R_{Ib}^2 + R_t^2 - 2R_t R_{Ib} \cos(\theta_t - \theta_{Ib}) \right]^{1/2}$$

$$dr = -\frac{R_t R_{Ib}}{r} \sin(\theta_t - \theta_{Ib}) d\theta_{Ib}$$

Limits of visibility

Limit of visibility	Range of independent variable θ_t			
	$\sin^{-1}\left(\frac{t}{R_t}\right) \leq \theta_t < (\theta_t)_{uc}$	$\theta_t = (\theta_t)_{uc}$	$(\theta_t)_{uc} < \theta_t \leq (\theta_t)_{lc}$	$(\theta_t)_{lc} < \theta_t \leq \frac{\pi}{2}$
Item numbers for limit equations				
$(\theta_{Ib})_l$	2	2	2	3
$(\theta_{Ib})_u$	5	6	7	7

1. $(\theta_t)_{lc}$ is the value of θ_t such that a tangent to the tube at this point will hit the point where inside fin 1 meets inside of the bumper

$$(\theta_t)_{lc} = \sin^{-1} \left[\sqrt{\frac{(R_{Ib}^2 - R_t^2)(R_{Ib}^2 - t^2)}{R_{Ib}^2}} + R_t t \right]$$

2. The lower limit is the minimum possible value of θ_{Ib} , that is, the point where inside fin 1 meets inside of bumper

$$(\theta_{Ib})_l = \sin^{-1} \left(\frac{t}{R_{Ib}} \right)$$

3. The lower limit is determined by a tangent to the tube

$$(\theta_{Ib})_l = \sin^{-1} \left[\frac{R_t \sin \theta_t - \cos \theta_t \sqrt{R_{Ib}^2 - R_t^2}}{R_{Ib}} \right]$$

4. $(\theta_t)_{uc}$ is the value of θ_t such that a tangent to the tube at this point will hit the inside of the bumper at $\theta_{Ib} = \pi/2$

$$(\theta_t)_{uc} = \frac{\pi}{2} - \cos^{-1} \left(\frac{R_t}{R_{Ib}} \right)$$

5. The upper limit is determined by a tangent to the tube and is less than 90°

$$(\theta_{Ib})_u = \delta$$

6. The upper limit is determined by a tangent to the tube and is equal to 90°

$$(\theta_{Ib})_u = \frac{\pi}{2}$$

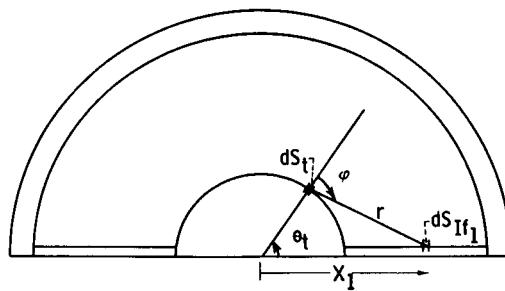
7. The upper limit is determined by a tangent to the tube and is greater than 90°

$$(\theta_{Ib})_u = \pi - |\delta|$$

For items 5, 6, and 7,

$$\delta = \sin^{-1} \frac{\sqrt{R_{Ib}^2 - R_t^2} \cos \theta_t + R_t \sin \theta_t}{R_{Ib}}$$

Case E2: Tube to Inside Fin 1



View-factor quantities

The view factor for this case can be obtained from Case A3 by making the following substitutions:

$$(\theta_l)_{A3} = (\theta_t)_{E2} \quad (R)_{A3} = (R_t)_{E2} \quad \beta = 0 \quad (t_l)_{A3} = (t)_{E2}$$

where the subscripts denote the case to which the symbol belongs. Then

$$a = X_l \sin \theta_t - t \cos \theta_t$$

$$da = \sin \theta_t dX_l$$

$$r = \left[X_l^2 + R_t^2 + t^2 - 2R_t(X_l \cos \theta_t + t \sin \theta_t) \right]^{1/2}$$

$$dr = \frac{X_l - R_t \cos \theta_t}{r} dX_l$$

Limits of visibility

Limit of visibility	Range of independent variable θ_t	
	$\sin^{-1}\left(\frac{t}{R_t}\right) \leq \theta_t < (\theta_t)_c$	$(\theta_t)_c \leq \theta_t \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(X_l)_l$	1	4
$(X_l)_u$	3	4

1. The lower limit is determined by a tangent to the tube

$$(X_l)_l = \frac{R_t - t \sin \theta_t}{\cos \theta_t}$$

2. $(\theta_t)_c$ is the value of θ_t such that a tangent to the tube at this point will hit the point where inside fin 1 meets inside of bumper; at values of θ_t larger than this, tube 1 occludes the view of inside fin 1

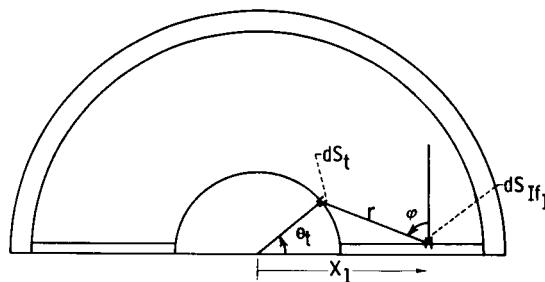
$$(\theta_t)_c = \sin^{-1} \left[\frac{\sqrt{(R_{Ib}^2 - R_t^2)(R_{Ib}^2 - t^2)} + R_t t}{R_{Ib}^2} \right]$$

3. The upper limit is the maximum possible value of X_1 , that is, the point where inside fin 1 meets inside of the bumper

$$(X_1)_u = R_{Tb}$$

4. Inside fin 1 is not visible from the tube for this range of θ_t .

Case E3: Inside Fin 1 to Tube



View-factor quantities

This case is related to Case E2 by

$$dS_{If_1} \, dF_{If_1-t} = dS_t \, dF_{t-If_1}$$

$$dF_{If_1-t} = \frac{R_t \, d\theta_t}{dX_1} \, dF_{t-If_1}$$

or

$$a = X_1 - R_t \cos \theta_t$$

$$da = R_t \sin \theta_t \, d\theta_t$$

$$r = \left[X_1^2 + R_t^2 + t^2 - 2R_t(t \sin \theta_t + X_1 \cos \theta_t) \right]^{1/2}$$

$$dr = \frac{R_t(X_1 \sin \theta_t - t \cos \theta_t)}{r} \, d\theta_t$$

Limits of visibility

Limit of visibility	Range of independent variable X_1
	$\sqrt{R_t^2 - t^2} \leq X_1 \leq \sqrt{R_{Tb}^2 - t^2}$
	Item numbers for limit equations
$(\theta_t)_l$	1
$(\theta_t)_u$	2

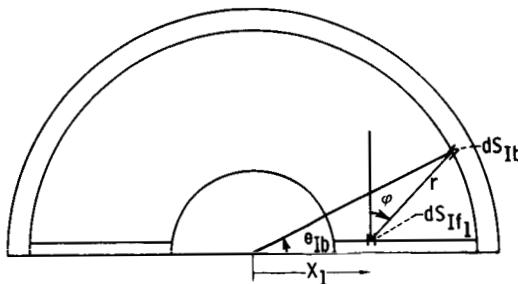
1. The lower limit is always the minimum possible value of θ_t , that is, the point where inside fin 1 meets the tube

$$(\theta_t)_l = \sin^{-1}\left(\frac{t}{R_t}\right)$$

2. The upper limit is determined by a tangent to the tube

$$(\theta_t)_u = \cos^{-1}\left(\frac{R_t X_1 - t \sqrt{X_1^2 + t^2 - R_t^2}}{X_1^2 + t^2}\right)$$

Case E4: Inside Fin 1 to Inside of Bumper



View-factor quantities

$$a = R_{Ib} \cos \theta_{Ib} - X_1$$

$$da = -R_{Ib} \sin \theta_{Ib} d\theta_{Ib}$$

$$r = [X_1^2 + t^2 + R_{Ib}^2 - 2R_{Ib}(X_1 \cos \theta_{Ib} + t \sin \theta_{Ib})]^{1/2}$$

$$dr = \frac{R_{Ib}X_1 \sin \theta_{Ib} - R_{Ib}t \cos \theta_{Ib}}{r} d\theta_{Ib}$$

Limits of visibility

Limit of visibility	Range of independent variable X_1		
	$\sqrt{R_t^2 - t^2} \leq X_1 < (X_1)_c$	$X_1 = (X_1)_c$	$(X_1)_c < X_1 \leq \sqrt{R_{Ib}^2 - t^2}$
Item numbers for limit equations			
$(\theta_t)_l$	1	1	1
$(\theta_{Ib})_u$	3	4	5

1. The lower limit is always the minimum possible value of θ_{Ib} , that is, the point where inside fin 1 meets inside of bumper

$$(\theta_{Ib})_l = \sin^{-1}\left(\frac{t}{R_{Ib}}\right)$$

2. $(x_1)_c$ is the value of x_1 such that a tangent to the tube from this point will hit the inside of the bumper at $\theta_{Ib} = \pi/2$

$$(x_1)_c = \frac{R_t(R_{Ib} - t)}{\sqrt{R_{Ib}^2 - R_t^2}}$$

3. The upper limit is determined by a tangent to the tube and is less than 90°

$$(\theta_{Ib})_u = \delta$$

4. The upper limit is determined by a tangent to the tube and is equal to 90°

$$(\theta_{Ib})_u = \frac{\pi}{2}$$

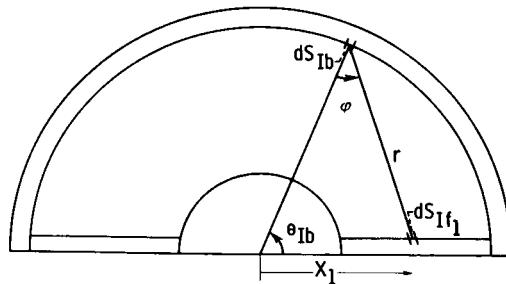
5. The upper limit is determined by a tangent to the tube and is greater than 90°

$$(\theta_{Ib})_u = \pi - |\delta|$$

For items 3, 4, and 5,

$$\delta = \sin^{-1} \left[\frac{R_t(tR_t + x_1 \sqrt{x_1^2 + t^2 - R_t^2}) + \sqrt{R_{Ib}^2 - R_t^2} (x_1 R_t - t \sqrt{x_1^2 + t^2 - R_t^2})}{R_{Ib}(x_1^2 + t^2)} \right]$$

Case E5: Inside of Bumper to Inside Fin 1



View-factor quantities

This case is related to Case E4 by

$$dS_{Ib} \, dF_{Ib-If_1} = dS_{If_1} \, dF_{If_1-Ib}$$

$$dF_{Ib-If_1} = \frac{dx_1}{R_{Ib} \, d\theta_{Ib}} \, dF_{If_1-Ib}$$

or

$$a = X_1 \sin \theta_{Ib} - t \cos \theta_{Ib}$$

$$da = \sin \theta_{Ib} \, dx_1$$

$$r = [R_{Ib}^2 + X_1^2 + t^2 - 2R_{Ib}(X_1 \cos \theta_{Ib} + t \sin \theta_{Ib})]^{1/2}$$

$$dr = \frac{X_1 - R_{Ib} \cos \theta_{Ib}}{r} \, dx_1$$

Limits of visibility

Limit of visibility	Range of independent variable θ_{Ib}	
	$\sin^{-1}\left(\frac{t}{R_{Ib}}\right) \leq \theta_{Ib} \leq (\theta_{Ib})_c$	$(\theta_{Ib})_c < \theta_{Ib} \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(X_1)_l$	2	3
$(X_1)_u$	4	4

1. $(\theta_{Ib})_c$ is the value of θ_{Ib} such that a tangent to the tube from this point will hit the tube at the point where inside fin 1 meets the tube

$$(\theta_{Ib})_c = \sin^{-1} \left[\frac{tR_t + \sqrt{(R_t^2 - t^2)(R_{Ib}^2 - R_t^2)}}{R_t R_{Ib}} \right]$$

2. The lower limit is the minimum possible value of X_1 , that is, the point where inside fin 1 meets the tube

$$(X_1)_l = \sqrt{R_t^2 - t^2}$$

3. The lower limit is determined by a tangent to the tube

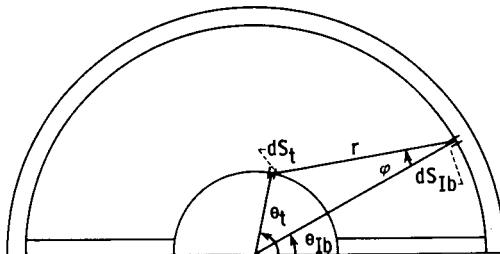
$$(x_1)_l = \frac{R_{Ib} R_t - t \left(R_t \sin \theta_{Ib} - \cos \theta_{Ib} \sqrt{R_{Ib}^2 - R_t^2} \right)}{R_t \cos \theta_{Ib} + \sin \theta_{Ib} \sqrt{R_{Ib}^2 - R_t^2}}$$

4. The upper limit is the maximum possible value of x_1 , that is, the point where inside fin 1 meets inside of bumper

$$(x_1)_u = \sqrt{R_{Ib}^2 - t^2}$$

It is possible that in some cases the dimensions of the configuration could be such that the tube could occlude the view of inside fin 1 for some values of θ_{Ib} . In order to determine if such is the case, compute the value of $(x_1)_c$ as given by item number 3 and call it $(x_1)_c$. If $(x_1)_c \geq \sqrt{R_{Ib}^2 - t^2}$ where $\sqrt{R_{Ib}^2 - t^2}$ is the maximum possible value of x_1 , then the tube occludes the view of inside fin 1 and $(x_1)_l = 0$ and $(x_1)_u = 0$. If $(x_1)_c < \sqrt{R_{Ib}^2 - t^2}$, however, then inside fin 1 is not occluded by the tube and the limits are those given in the table.

Case E6: Inside of Bumper to Tube 1



View-factor quantities

This case is related to Case E1 by

$$dS_{Ib} dF_{Ib-t} = dS_t dF_{t-Ib}$$

$$dF_{Ib-t} = \frac{R_t}{R_{Ib}} \frac{d\theta_t}{d\theta_{Ib}} dF_{t-Ib}$$

or

$$a = R_t \sin(\theta_t - \theta_{Ib})$$

$$da = R_t \cos(\theta_t - \theta_{Ib}) d\theta_t$$

$$r = [R_t^2 + R_{Ib}^2 - 2R_t R_{Ib} \cos(\theta_t - \theta_{Ib})]^{1/2}$$

$$dr = \frac{R_t R_{Ib} \sin(\theta_t - \theta_{Ib})}{r} d\theta_t$$

Limits of visibility

Limit of visibility	Range of independent variable θ_{Ib}			
	$\sin^{-1}\left(\frac{t}{R_{Ib}}\right) \leq \theta_{Ib} < (\theta_{Ib})_{uc}$	$\theta_{Ib} = (\theta_{Ib})_{uc}$	$(\theta_{Ib})_{uc} < \theta_{Ib} \leq (\theta_{Ib})_{lc}$	$(\theta_{Ib})_{lc} < \theta_{Ib} \leq \frac{\pi}{2}$
Item numbers for limit equations				
$(\theta_t)_l$	2	2	2	3
$(\theta_t)_u$	5	6	7	7

1. $(\theta_{Ib})_{lc}$ is the value of θ_{Ib} such that a tangent to the tube from this point will be tangent at the point where inside fin 1 meets the tube

$$(\theta_{Ib})_{lc} = \sin^{-1} \left[\frac{R_t t + \sqrt{(R_t^2 - t^2)(R_{Ib}^2 - R_t^2)}}{R_t R_{Ib}} \right]$$

2. The lower limit is the minimum possible value of θ_t , that is, the point where inside fin 1 meets the tube

$$(\theta_t)_l = \sin^{-1}\left(\frac{t}{R_t}\right)$$

3. The lower limit is determined by a tangent to the tube

$$(\theta_t)_l = \sin^{-1} \left[\frac{R_t \sin \theta_{Ib} - \sqrt{R_{Ib}^2 - R_t^2} \cos \theta_{Ib}}{R_{Ib}} \right]$$

4. $(\theta_{Ib})_{uc}$ is the value of θ_{Ib} such that a tangent to the tube from this point will hit the tube at $\theta_t = \pi/2$

$$(\theta_{Ib})_{uc} = \sin^{-1}\left(\frac{R_t}{R_{Ib}}\right)$$

5. The upper limit is determined by a tangent to the tube and is less than 90°

$$(\theta_t)_u = |\delta|$$

6. The upper limit is determined by a tangent to the tube and is equal to 90°

$$(\theta_t)_u = \frac{\pi}{2}$$

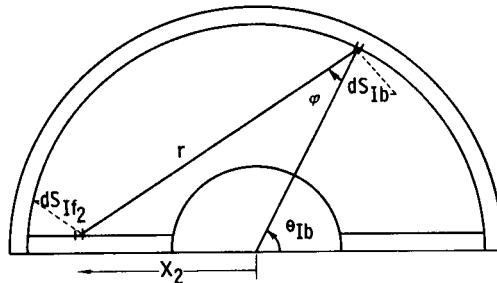
7. The upper limit is determined by a tangent to the tube and is greater than 90°

$$(\theta_t)_u = \pi - |\delta|$$

For items 5, 6, and 7,

$$\delta = \sin^{-1} \left(\frac{R_t \sin \theta_{Ib} + \sqrt{R_{Ib}^2 - R_t^2} \cos \theta_{Ib}}{R_{Ib}} \right)$$

Case E7: Inside of Bumper to Inside Fin 2



View-factor quantities

$$a = X_2 \sin \theta_{Ib} + t \cos \theta_{Ib}$$

$$da = \sin \theta_{Ib} dx_2$$

$$r = \left[R_{Ib}^2 + X_2^2 + t^2 - 2R_{Ib}(t \sin \theta_{Ib} - X_2 \cos \theta_{Ib}) \right]^{1/2}$$

$$dr = \frac{X_2 + R_{Ib} \cos \theta_{Ib}}{r} dx_2$$

Limits of visibility

Limit of visibility	Range of independent variable θ_{Ib}	
	$\sin^{-1} \left(\frac{t}{R_{Ib}} \right) \leq \theta_{Ib} \leq (\theta_{Ib})_c$	$(\theta_{Ib})_c < \theta_{Ib} \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(x_2)_l$	1	3
$(x_2)_u$	1	4

1. Inside fin 2 is not visible from the inside of the bumper for this range of θ_{Ib} .

2. $(\theta_{Ib})_c$ is the value of θ_{Ib} such that a tangent to the tube from this point will hit the point where inside fin 2 meets inside of the bumper; at values of θ_{Ib} less than this, tube 1 occludes the view of inside fin 2

$$(\theta_{Ib})_c = \sin^{-1} \left[\frac{t(2R_t^2 - R_{Ib}^2) + 2R_t \sqrt{(R_{Ib}^2 - t^2)(R_{Ib}^2 - R_t^2)}}{R_{Ib}^3} \right]$$

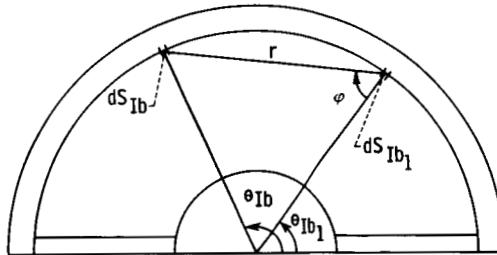
3. The lower limit is determined by a tangent to the tube

$$(x_2)_l = \frac{R_t R_{Ib} - t \left(\sqrt{R_{Ib}^2 - R_t^2} \cos \theta_{Ib} + R_t \sin \theta_{Ib} \right)}{\sqrt{R_{Ib}^2 - R_t^2} \sin \theta_{Ib} - R_t \cos \theta_{Ib}}$$

4. The upper limit is the maximum possible value of x_2 , that is, the point where inside fin 2 meets inside of bumper

$$(x_2)_u = \sqrt{R_{Ib}^2 - t^2}$$

Case E8: Inside Bumper to Inside Bumper



This is the only case considered thus far in which a surface can view itself. To avoid confusion in terminology, the independent variable will be subscripted for this case only.

View-factor quantities

$$a = R_{Ib} \sin(\theta_{Ib} - \theta_{Ib1})$$

$$da = R_{Ib} \cos(\theta_{Ib} - \theta_{Ib1}) d\theta_{Ib}$$

$$r = \left\{ 2R_{Ib}^2 [1 - \cos(\theta_{Ib} - \theta_{Ib1})] \right\}^{1/2}$$

$$dr = \frac{R_{Ib}^2 \sin(\theta_{Ib} - \theta_{Ib1})}{r} d\theta_{Ib}$$

Limits of visibility

Limit of visibility	Range of independent variable θ_{Ib1}	
	$\sin^{-1}\left(\frac{t}{R_{Ib}}\right) \leq \theta_{Ib1} < (\theta_{Ib1})_c$	$(\theta_{Ib1})_c \leq \theta_{Ib1} \leq \frac{\pi}{2}$
Item numbers for limit equations		
$(\theta_{Ib})_l$	1	1
$(\theta_{Ib})_u$	3	4

1. The lower limit is the minimum possible value of θ_{Ib} , that is, the point where inside fin 1 meets inside of bumper

$$(\theta_{Ib})_l = \sin^{-1}\left(\frac{t}{R_{Ib}}\right)$$

2. $(\theta_{Ib1})_c$ is the value of θ_{Ib1} such that a tangent to the tube from this point will hit the point where inside fin 2 meets inside of bumper

$$(\theta_{Ib1})_c = \sin^{-1}\left[\frac{2R_t \sqrt{(R_{Ib}^2 - R_t^2)(R_{Ib}^2 - t^2)} + t(2R_t^2 - R_{Ib}^2)}{R_{Ib}^3}\right]$$

3. The upper limit is determined by a tangent to the tube

$$(\theta_{Ib})_u = \theta_{Ib1} + 2 \cos^{-1}\left(\frac{R_t}{R_{Ib}}\right)$$

4. The upper limit is the maximum possible value of θ_{Ib} , that is, the point where inside fin 2 meets inside of bumper

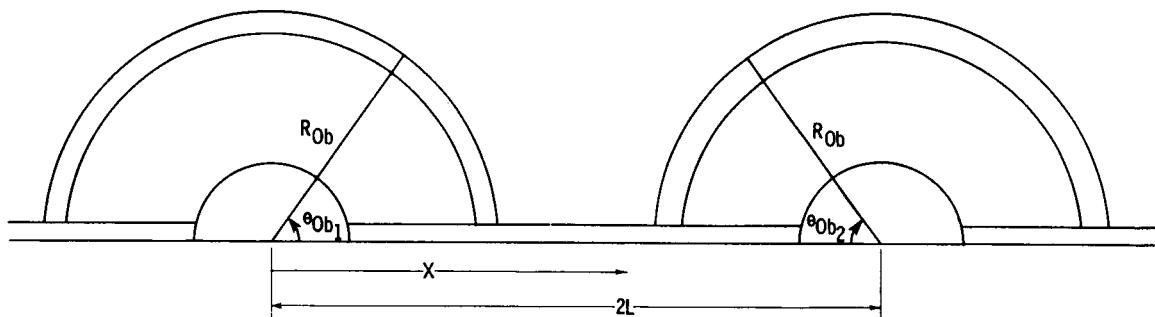
$$(\theta_{Ib})_u = \pi - \sin^{-1}\left(\frac{t}{R_{Ib}}\right)$$

It is possible that in some cases the dimensions of the configuration could be such that the tube could interfere with the lower limit of visibility. In order to determine if such is the case, compute the following value of $(\theta_{Ib})_c$:

$$(\theta_{Ib})_c = \theta_{Ib1} - 2 \cos^{-1}\left(\frac{R_t}{R_{Ib}}\right)$$

If $(\theta_{Ib})_c \leq \sin^{-1}(t/R_{Ib})$ where $\sin^{-1}(t/R_{Ib})$ is the minimum possible value of θ_{Ib} , then the tube does not interfere with the lower limit and $(\theta_{Ib})_l$ is given by item number 1. If $(\theta_{Ib})_c > \sin^{-1}(t/R_{Ib})$, however, then the lower limit is determined by a tangent to the tube and $(\theta_{Ib})_l = (\theta_{Ib})_c$.

Cases E9, E10, E11, and E12



The view factors and limits of visibility for these cases can be obtained from the corresponding cases of configuration A' by making the following substitutions:

$$(\theta_1)_{A'} = (\theta_{Ob1})_E \quad (R)_{A'} = (R_{Ob})_E$$

$$(\theta_2)_{A'} = (\theta_{Ob2})_E \quad (x_1)_{A'} = (x)_E$$

where the subscripts denote the case to which the symbol belongs. Then

Case E9: Outside bumper 1 to outside bumper 2 (use Case A'1).

Case E10: Outside bumper 1 to outside fin (use Case A'3).

Case E11: Outside fin to outside bumper 1 (use Case A'4).

Case E12: Outside fin to outside bumper 2 (use Case A'5).

CONCLUDING REMARKS

The role of the view factor in radiation heat-transfer calculations is discussed. A general formula is derived for the view factor between finitely or infinitely long differential strips of general cylindrical surfaces having parallel generating lines. The view-factor formula for finite length is not significantly more complicated than that for infinite length and can be just

as readily used. View factors obtained by the general formula are presented for various pairs of differential strips of six different systems of circular cylinders and planes characteristic of radiator fin-tube geometries. The limits of visibility from each surface to all visible surfaces of each system are also presented.

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National Aeronautics and Space Administration
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APPENDIX - SYMBOLS

A	area
a	projection of r onto plane perpendicular to normal to strip l
B	radiosity, total radiant energy per unit area and per unit time leaving surface
F	view factor, fraction of total radiant energy that leaves one surface and strikes another
dF	view factor between differential elements
g	incident radiant energy per unit area and per unit time on surface from external sources, such as sun or planets
H	total incident energy per unit area and per unit time on surface
i	index of surface
j	index of increment of surface
L	half distance between tube centers for configurations A, A', and E; distance between tube center and bumper for configurations B, C, and D
n	number of surfaces visible to certain surface
R	tube radius for configurations A to E
\mathcal{R}	distance between differential elements dA_1 and dA_2
r	distance between differential strips
S	general surface; distance between tube centers for configuration B
S^*	particular point on surface
T	temperature
t	fin half thickness for configurations A to E
t_l	mid-fin half thickness for configuration A
t_o	fin half thickness at tube for configuration A
W	lower strut coordinate for configurations B and C
X	fin coordinate for configurations A, A', and E; upper strut coordinate for configurations B, C, and D
Y	bumper coordinate for configurations B, C, and D

Z longitudinal length of radiator segment
 β taper angle of fin for configuration A
 ϵ emittance
 θ tube coordinate for configurations A to E
 χ angle from line normal to area element 1 to line joining this element and area element 2
 ρ reflectivity
 σ Stefan-Boltzmann constant
 ϕ angle between normal to strip 1 and shortest line joining strip 1 and strip 2
 ψ angle between r and \mathbf{R}
 $d\omega$ solid angle subtended at dA_1 by dA_2

Subscripts:

dA differential element of area
 b bumper
 c critical value
 f fin
 i surface i
 I_b inside cylindrical bumper, configuration E
 I_f inside fin
 j increment j
 l lower limit of visibility or integration
 l_c critical value on lower limit
 l_s lower strut, configurations B and C
 m_c middle critical value
 O_b outside cylindrical bumper, configuration E
 t tube, configurations A to E

u upper limit of visibility or integration
uc critical value on upper limit
us upper strut, configurations B, C, and D
1 surface 1
2 surface 2
3 surface 3

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